COMP718: Ontologies and Knowledge Bases

Lectures 9 and 10: ontology-based data access

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The following slides are heavily based on David Toman's slides of his seminar at UKZN d.d. 29-3-2011; slides used with permission

Ontology-Based Data Access: Options

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- E. Franconi and G. Weddell

Queries and Ontologies

Ontology-based Data Access

Enriches query answers over *explicitly represented data* using *background knowledge* (captured using an *ontology*.)

Example

- Bob is a BOSS (explicit data)
- Every BOSS is an EMPloyee (ontology)

List all EMPloyees $\Rightarrow$ $\{$Bob$\}$ (query)

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Setup

$$(\mathcal{A}, \mathcal{T}) \xrightarrow{\quad Q \quad} \mathcal{A}'$$

- $\mathcal{A}$ "the data" — set of *ground tuples*: $BOSS(Bob)$
- $\mathcal{T}$ "the knowledge" — FO[†] sentences: $\forall x.BOSS(x) \to EMP(x)$
- Q "the question" — a FO[†] formula: $EMP(x)$

[†] or an appropriate fragment of FO

What is this good for?

1. Enriches explicit data with background knowledge
2. Physical Data Independence

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Interpretations and Models, Data and Queries

Interpretation $\mathcal{I}$:

- A Domain Δ of *objects*
- An Interpretation Function $(\cdot)^{\mathcal{I}}$ that maps

  constants to objects and predicates to sets of tuples of objects

Models

A *model* of a *formula (set of formulas)* is an interpretation that makes the formula (all formulas in the set) true.

What does $\mathcal{A} = \{\mathrm{Emp}(\mathit{Bob}), \mathrm{Emp}(\mathit{Sue})\}$ mean?

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How do we Answer Queries: The Simple Answer

Logical Implication

A *set of formulas* entails ($\models$) *another formula* if every *model* of the former is also model of the later.

Definition (Query Answering)

$$Q(\mathcal{A},\mathcal{T}) = \{\vec{a} \mid \mathcal{T} \cup \mathcal{A} \models Q[\vec{a}]\}$$

Operationally (with *standard names*):

$$Q(\mathcal{A},\mathcal{T}) = \bigcap_{\mathcal{I} \models \mathcal{T} \cup \mathcal{A}} Q(\mathcal{I})$$

↑ this is a problem

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Running rather Slowly, Eh?

Example

- relations: "$ColNode(x, y)$" and "$Edge(x, y)$";
- ontology: $\forall x.Node(x) \rightarrow \exists y.ColNode(x, y)$,
  $\forall x, y.ColNode(x, y) \rightarrow Colour(y)$;
- the data: a graph $(Node^{\mathcal{I}}, Edge^{\mathcal{I}})$, and
  $Colour^{\mathcal{I}} = \{r, g, b\}$.

What does the following query say?

$$\exists x, y, z.Edge(x, y) \wedge ColNode(x, z) \wedge ColNode(y, z)$$

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The KB has TOO MANY MODELS (so we have to look at many)

1. $(\mathcal{T}, \mathcal{A})$ have exactly one model $\mathcal{I}$: then $Q(\mathcal{A}, \mathcal{T}) = Q(\mathcal{I})$

   ... this is how *people will think about query answering* anyway!

2. $(\mathcal{T}, \mathcal{A})$ have many models, say $\mathcal{I}_j$ $(j \in J)$:

   Option I: restrict $\mathcal{T}$ to make it feasible: *(simple) Horn theories*

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   $\Rightarrow$ e.g., safe queries in Codd's relational model

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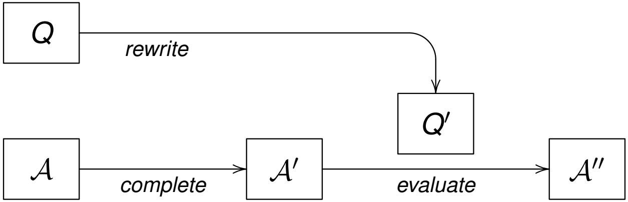

v1.0: *rewrite*: incorporate $\mathcal{T}$ into Q,
complete: an identity ($\mathcal{A}' = \mathcal{A}$)

...[Calvanese et al.]

v2.0: *rewrite*: rewrite independently of $\mathcal{T} \cup \mathcal{A}$,
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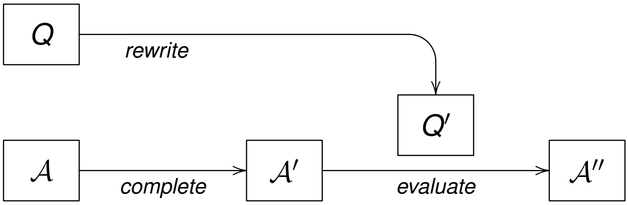

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How to make $\mathcal{T}$ Easy?

Definition (DL-Lite$_{\text{horn}}$)

roles: $R ::= P \mid P^-$, concepts: $C ::= \bot \mid A \mid \exists R$.

1. An *ontology (TBox)* is a finite set $\mathcal{T}$ of *concept* inclusions $C_1 \sqcap \cdots \sqcap C_n \sqsubseteq C$;
2. The *Data (ABox)* is a finite set $\mathcal{A}$ of *concept* and *role* assertions $C(a)$ and $R(a, b)$;
3. A Conjunctive Query (CQ):
   an existentially quantified finite conjunction of atoms.

The Master Plan (v1.0)

IDEA:

1. Incorporate the background knowledge (i.e., $\mathcal{T}$) into the *query*.
2. Use the *rewritten query* against the ABox $\mathcal{A}$

$\Rightarrow$ and use a relational system to do this *efficiently*.

Example

$\mathcal{T} = \{EMP \sqsubseteq \exists MANAGES, \exists MANAGES^{-} \sqsubseteq BOSS, BOSS \sqsubseteq EMP\}$

$\mathcal{A} = \{BOSS(Bob), EMP(Sue)\}$

$$Q(x,z) \leftarrow \exists y.MANAGES(x,y) \land MANAGES(z,y)$$
$$Q(x,x) \leftarrow \exists y.MANAGES(x,y) \quad (factor)$$
$$Q(x,x) \leftarrow EMP(x) \quad (T1)$$
$$Q(x,x) \leftarrow BOSS(x) \quad (T3)$$

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The Master Plan (v2.0)

IDEA:

1. Incorporate the background knowledge (i.e., $\mathcal{T}$) into the *data*.

   $\Rightarrow$ make *implicit knowledge explicit (data completion)*.
2. Use the *data completion* (only) to answer queries

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Issues:

1. How to complete the data?

   Naive *unfolding* of $\mathcal{T}$: large/infinite (due to existentials)

   $\Rightarrow$ we define a *canonical interpretation* $\mathcal{I}_{\mathcal{K}}$ (representatives)
2. Can we then use the original Conjunctive Query?

   Not directly: $Q(\mathcal{I}_{\mathcal{K}})$ can produce *"spurious matches"*

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$\mathcal{T} = \{BOSS \sqsubseteq EMP\}$, $\mathcal{A} = \{BOSS(Bob)\}$, $Q \equiv EMP(x)$

1. $\mathcal{I}_{\mathcal{K}} = \{BOSS(Bob), EMP(Bob)\}$ (data completion)
2. $Q(\mathcal{I}_{\mathcal{K}}) = \{Bob\}$ (relational query)

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Canonical Interpretations

ABox completion: the Canonical Interpretation $\mathcal{I}_\mathcal{K}$

$$A^{\mathcal{I}_\mathcal{K}} = \{a \in \mathrm{Ind}(\mathcal{A}) \mid \mathcal{K} \models A(a)\} \cup \{c_R \in \Delta^{\mathcal{I}_\mathcal{K}} \mid \mathcal{T} \models \exists R^- \sqsubseteq A\},$$

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. . . c_R's only used "when necessary" (for *generating* roles)

Lemma

There are queries

- $q_A^\mathcal{T}$ *s.t.* $\mathrm{ans}(q_A^\mathcal{T}, \mathcal{A}) = A^{\mathcal{I}_\mathcal{K}}$, *and*
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for every KB $(\mathcal{T}, \mathcal{A})$ *and primitive concept A and role P.*

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$$\mathcal{A} = \{BOSS(Bob), EMP(Sue)\}$$

Then $EMP^{\mathcal{I}_\mathcal{K}} = \{Bob, Sue,$  $\}$, $BOSS^{\mathcal{I}_\mathcal{K}} = \{Bob,$  $\}$, and
$MANAGES^{\mathcal{I}_\mathcal{K}} = \{(Bob,$  $), (Sue,$  $), ($  $,$  $)\}$.

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for every KB $(\mathcal{T}, \mathcal{A})$ *and primitive concept A and role P.*

free consistency test: $q_\perp^\mathcal{T}(\mathcal{A}) = \emptyset$

Canonical Interpretations

ABox completion: the Canonical Interpretation $\mathcal{I}_{\mathcal{K}}$

$$A^{\mathcal{I}_{\mathcal{K}}} = \{a \in \mathrm{Ind}(\mathcal{A}) \mid \mathcal{K} \models A(a)\} \cup \{c_R \in \Delta^{\mathcal{I}_{\mathcal{K}}} \mid \mathcal{T} \models \exists R^- \sqsubseteq A\},$$

$$P^{\mathcal{I}_{\mathcal{K}}} = \{(a, b) \in \mathrm{Ind}(\mathcal{A}) \times \mathrm{Ind}(\mathcal{A}) \mid P(a, b) \in \mathcal{A}\} \cup$$
$$\{(d, c_P) \in \Delta^{\mathcal{I}_{\mathcal{K}}} \times \mathsf{N}_\mathsf{I}^{\mathcal{T}} \mid d \rightsquigarrow c_P\} \cup \{(c_{P^-}, d) \in \mathsf{N}_\mathsf{I}^{\mathcal{T}} \times \Delta^{\mathcal{I}_{\mathcal{K}}} \mid d \rightsquigarrow c_{P^-}\}$$

... c_R's only used "when necessary" (for *generating* roles)

Lemma

There are queries

- $q_A^{\mathcal{T}}$ *s.t.* $\mathrm{ans}(q_A^{\mathcal{T}}, \mathcal{A}) = A^{\mathcal{I}_{\mathcal{K}}}$, *and*
- $q_P^{\mathcal{T}}$ *s.t.* $\mathrm{ans}(q_P^{\mathcal{T}}, \mathcal{A}) = P^{\mathcal{I}_{\mathcal{K}}}$

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Query Rewriting (TBox-free)

Example

$\mathcal{T} = \{EMP \sqsubseteq \exists MANAGES, \exists MANAGES^{-} \sqsubseteq BOSS, BOSS \sqsubseteq EMP\}$

$\mathcal{A} = \{EMP(Bob), EMP(Sue)\}$

Queries:

1. $\exists v.MANAGES(v, v)$
2. $\exists y.MANAGES(x, y) \wedge MANAGES(z, y)$

Query Rewriting

$$\exists \bar{u}.\varphi \quad \mapsto \quad \exists \bar{u}.\varphi \wedge \varphi_1 \wedge \varphi_2 \wedge \varphi_3$$

where
- φ_1 eliminates answers containing c_R's;
- φ_2 eliminates problem (1) above; and
- φ_3 eliminates problem (2) above.

} *selections* in SQL

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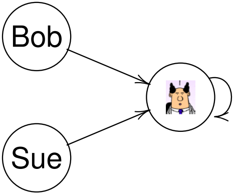
$\mathcal{I}_{\mathcal{K}}$:



$Q_1(\mathcal{I}_{\mathcal{K}}) = \text{true}$
$Q_2(\mathcal{I}_{\mathcal{K}}) = \{(Bob, Sue)\}$

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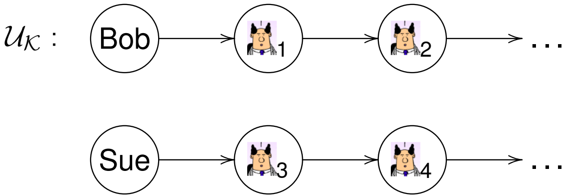

$Q_1(\mathcal{U}_{\mathcal{K}}) = \text{false}$
$Q_2(\mathcal{U}_{\mathcal{K}}) = \{\}$

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v1.0 vs. v2.0

	v1.0 (query rewriting)	v2.0 (data completion)
Queries	rewriting is exponential in $\|Q\|$	data only grows polynomially in $\|\mathcal{A}\|$
Updates	applies to original data	needs rematerialize data completion

Option II: Exact Answers

IDEA:

Restrict *queries* to those

whose answer does NOT depend on the choice of model of $\mathcal{T} \cup \mathcal{A}$:

$$\textit{for all } \mathcal{I}, \mathcal{J} \models \mathcal{T} \cup \mathcal{A} \textit{ we have } Q(\mathcal{I}) = Q(\mathcal{J})$$

In practice—given $\mathcal{T}$, Q, and *FIXED signature for $\mathcal{A}$*:

$$\textit{for all } \mathcal{I}, \mathcal{J} \models \mathcal{T} \cup \mathcal{A} \textit{ we have } Q(\mathcal{I}) = Q(\mathcal{J}) \qquad (*)$$

for *every choice of $\mathcal{A}$ over the FIXED signature.*

Advantages: no restrictions of $\mathcal{T}$ and Q

(modulo deciding whether the condition $(*)$ holds)

Issues: how does this help us??

a FO rewriting *over $\mathcal{A}$* exists $\Rightarrow$ a relational query

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Beth Definability and Interpolation

How do we test for $(*)$?

Beth Definability

Q satisfies $(*)$ if

$$\mathcal{T} \cup \mathcal{T}' \models Q \rightarrow Q'$$

where $\mathcal{T}'$ (Q') is $\mathcal{T}$ (Q) in which symbols *NOT in $\mathcal{A}$* are *primed*.

. . . this only works under CWA!

How do we rewrite Q?

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$\models \varphi \rightarrow \psi$ then $\models \varphi \rightarrow \gamma \rightarrow \psi$,

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  ⇒ Information Integration

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References

Option I, v1.0: D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati. Tractable reasoning and efficient query answering in description logics: The DL-Lite family. J. of Automated Reasoning, 39(3):385-429, 2007.

Option I, v2.0: C. Lutz, D. Toman, and F. Wolter. Conjunctive query answering in the description logic EL using a relational database system. In Proc. IJCAI, 2070-2075, 2009.

R. Kontchakov, C. Lutz, D. Toman, F. Wolter, and M. Zakharyaschev. The combined approach to query answering in DL-Lite. In Proc. KR, 2010.

Option II: D. Toman and G. Weddell. Fundamentals of Physical Design and Query Compilation. Morgan and Claypool, Synthesis lectures, *Data Management Series*. 2011.