Some questions and problems (not exhaustive...)

- Is Tshwane a more specific instance of Gauteng, or a part of it?
- Is a tunnel part of the mountain?
- What is the difference, if any, between how Cell nucleus and Cell are related and how Receptor and Cell wall are related?
- And w.r.t. Brain part of Human and versus Hand part of Boxer? (assuming boxers must have their own hands)
- A classical example: hand is part of musician, musician part of orchestra, but clearly, the musician’s hands are not part of the orchestra. Is part-of then not transitive, or is there a problem with the example?

Analysis of the issues from diverse angles

- Mereological theories (Varzi, 2004), usage & extensions (e.g. mereotopology, relation with granularity, set theory)
- Early attempts with direct parthood, SEP triples, and other outstanding issues, some still remaining
- Cognitive & linguistic issues from meronymy
- Usage in conceptual modelling and ontology engineering
- Subject domains: thus far, mainly geo, bio, medicine
Ground Mereology

Reflexivity (everything is part of itself)

\[ \forall x \left( \text{part of}(x, x) \right) \] (1)

Antisymmetry (two distinct things cannot be part of each other, or: if they are, then they are the same thing)

\[ \forall x, y \left( \left( \text{part of}(x, y) \land \text{part of}(y, x) \right) \rightarrow x = y \right) \] (2)

Transitivity (if \( x \) is part of \( y \) and \( y \) is part of \( z \), then \( x \) is part of \( z \))

\[ \forall x, y, z \left( \left( \text{part of}(x, y) \land \text{part of}(y, z) \right) \rightarrow \text{part of}(x, z) \right) \] (3)

Proper parthood

\[ \forall x, y \left( \text{proper part of}(x, y) \equiv \text{part of}(x, y) \land \neg \text{part of}(y, x) \right) \] (4)

Proper parthood

\[ \forall x, y \left( \text{proper part of}(x, y) \equiv \text{part of}(x, y) \land \neg \text{part of}(y, x) \right) \] (5)

Asymmetry (if \( x \) is part of \( y \) then \( y \) is not part of \( x \))

\[ \forall x, y \left( \text{part of}(x, y) \rightarrow \neg \text{part of}(y, x) \right) \] (6)

Irreflexivity (\( x \) is not part of itself)

\[ \forall x \neg \left( \text{part of}(x, x) \right) \] (7)

Defining other relations with \text{part of}

Overlap (\( x \) and \( y \) share a piece \( z \))

\[ \forall x, y \left( \text{overlap}(x, y) \equiv \exists z \left( \text{part of}(z, x) \land \text{part of}(z, y) \right) \right) \] (8)

Underlap (\( x \) and \( y \) are both part of some \( z \))

\[ \forall x, y \left( \text{underlap}(x, y) \equiv \exists z \left( \text{part of}(z, x) \land \text{part of}(z, y) \right) \right) \] (9)

Over- & undercross (over/underlap but not part of)

\[ \forall x, y \left( \text{overcross}(x, y) \equiv \text{overlap}(x, y) \land \neg \text{part of}(x, y) \right) \] (10)

\[ \forall x, y \left( \text{undercross}(x, y) \equiv \text{underlap}(x, y) \land \neg \text{part of}(y, x) \right) \] (11)

Proper overlap & Proper underlap

\[ \forall x, y \left( \text{p_overlap}(x, y) \equiv \text{overlap}(x, y) \land \text{overlap}(y, x) \right) \] (12)

\[ \forall x, y \left( \text{p_underlap}(x, y) \equiv \text{underlap}(x, y) \land \text{underlap}(y, x) \right) \] (13)

With \( x \) as part, what to do with the remainder that makes up \( y \)?

- Weak supplementation: every proper part must be supplemented by another, disjoint, part. MM
- Strong supplementation: if an object fails to include another among its parts, then there must be a remainder. EM

Problem with EM: non-atomic objects with the same proper parts are identical, because of this (extensionality principle), but sameness of parts may not be sufficient for identity. E.g.: two objects can be distinct purely based on arrangement of its parts, differences statue and its marble (multiplicative approach)
General Extentional Mereology

- Strong supplementation [EM]
  \[-\text{part} \_ \text{of}(y, x) \rightarrow \exists z (\text{part} \_ \text{of}(z, y) \land \neg \text{overlap}(z, x))\] (14)
- And add unrestricted fusion [GEM]. Let \(\phi\) be a property or condition, then for every satisfied \(\phi\) there is an entity consisting of all entities that satisfy \(\phi\). \(^1\) Then:
  \[\exists x \phi \rightarrow \exists z \forall y (\text{overlap}(y, z) \leftrightarrow \exists x (\phi \land \text{overlap}(y, x)))\] (15)
- Note that in EM and upward we have identity, from which one can prove acyclicity for ppo
- There are more mereological theories, and the above is not uncontested (more about that later)

\(^1\) Need to refer to classes, but desire to stay within FOL. Solution: axiom schema with only predicates or open formulas

Can any of this be represented in a decidable fragment of first order logic for use in ontologies and (scalable) software implementations?

Relations between common mereological theories

Fig. 1: Hasse diagram of mereological theories; from weaker to stronger, going uphill (after [44]).

- Early days (90s) and simplest options: DL-role \(R\) as partof, or has-part added as primitive role as \(\succeq\), model it as the transitive closure of a parthood relation (16) and define e.g. Car as having wheels that in turn have tires (17):
  \[\succeq \equiv (\text{primitive-part})^*\] (16)
  \[\text{Car} \equiv \exists \exists \succeq .(\text{Wheel} \sqcap \exists \ge .\text{Tire})\] (17)
  Then Car \(\sqsubseteq \exists \ge .\text{Tire}
- SEP triples with \(\mathcal{ALC}\)
- What \(\mathcal{SHIQ}\) fixes cf. \(\mathcal{ALC}\): Transitive roles, Inverse roles (to have both part-of and has-part), Role hierarchies (e.g. for subtypes of part-of), qualified Number restrictions (e.g. to represent that a bycicle has-part 2 wheels)
- Build-your-own DL-language
Table: Properties of parthood and proper parthood compared to their support in \( DC^{\mu}, SHOIN \) and \( SROIQ \). *: properties of the parthood relation (in M); †: properties of the proper parthood relation (in M).

<table>
<thead>
<tr>
<th>Feature</th>
<th>( DC^{\mu} )</th>
<th>( SHOIN )</th>
<th>( SROIQ )</th>
<th>DL-Lite_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexivity</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Antisymmetry</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Transitivity</td>
<td>†</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>†</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Irreflexivity</td>
<td>†</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Acyclicity</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Need to commit to a foundational ontology. Recently, linked to BFO http://obofoundry.org/ro/#mappings (test release)

Linguistic use of part-whole relations

- Class-level relations
  - \( C \text{ part}\_\text{of} \ C_1 \triangleq \) for all \( c, t \), if \( Cct \) then there is some \( c_1 \) such that \( C_1c_1t \) and \( c \text{ part}\_\text{of} \ C_1 \at t \).
  - \( P \text{ part}\_\text{of} \ P_1 \triangleq \) for all \( p \), if \( Pp \) then there is some \( p_1 \) such that: \( P_1p_1 \) and \( p \text{ part}\_\text{of} \ P_1 \).
  - \( C \text{ contained}\_\text{in} \ C_1 \triangleq \) for all \( c, t \), if \( Cct \) then there is some \( c_1 \) such that: \( C_1c_1t \) and \( c \text{ contained}\_\text{in} \ C_1 \at t \)

- Instance-level relations
  - \( c \text{ part}\_\text{of} \ C_1 \at t \) - a primitive relation between two continuant instances and a time at which the one is part of the other
  - \( p \text{ part}\_\text{of} \ P_1, r \text{ part}\_\text{of} \ R_1 \) - a primitive relation of parthood, holding independently of time, either between process instances (one a subprocess of the other), or between spatial regions (one a subregion of the other)
  - \( c \text{ contained}\_\text{in} \ C_1 \at t \triangleq c \text{ located}\_\text{in} \ C_1 \at \ t \) and not \( c \text{ overlap} \ C_1 \at \ t \)
  - \( c \text{ located}\_\text{in} \ r \at t \) - a primitive relation between a continuant instance, a spatial region which it occupies, and a time

- Part of?
  - * Centimeter part of Decimeter
  - * Decimeter part of Meter
  — therefore Centimeter part of Meter
  - * Meter part of SI
  — but not Centimeter part of SI

- Transitivity?
  - * Person part of Organisation
  — Organisation located in Bolzano
  — therefore Person located in Bolzano?
  — but not Person part of Bolzano
**Meronymy**

**Linguistic use of part-whole relations (meronymy)**

- **Part of?**
  - Centimeter part of Decimeter
  - Decimeter part of Meter
  - *therefore* Centimeter part of Meter
  - Meter part of SI
  - *but not* Centimeter part of SI

- **Transitivity?**
  - Person member of Organisation
  - Organisation located in Bolzano
  - *therefore* Person located in Bolzano?
  - *but not* Person member of Bolzano

**Addressing the issues**

- **Which part of?**
  - CellMembrane structural part of RedBloodCell
  - RedBloodCell part of Blood
  - *but not* CellMembrane structural part of Blood
  - Receptor structural part of CellMembrane
  - *therefore* Receptor structural part of RedBloodCell

**Efforts to disambiguate this confusion; e.g. an informal taxonomy by Winston et al (1987), list of 6 types motivated by UML conceptual modeling (Odell) ontology-inspired conceptual modelling (Guizzardi)**

**Location, containment, membership of a collective, quantities of a mass**

**Relatively well-settled debate on transitivity, or not**
Overview

- Mereological `part_of` (and subtypes) versus 'other' part-whole relations
- Categories of object types of the part-whole relation changes
- Structure these relations by (non/in)transitivity and kinds of relata
- Simplest mereological theory, M.
- Commit to a foundational ontology: DOLCE (though one also could choose, a.o., BFO, OCHRE, GFO, ...)

The taxonomy

Part-whole relations

```
∀x, y (member_of(x, y) ≡ mpart_of(x, y) ∧ (POB(x) ∨ SOB(x)) ∧ SOB(y))
```

```
∀x, y (constitutes(x, y) ≡ constituted_of(y, x) ≡ mpart_of(x, y) ∧ POB(y) ∧ M(x))
```
Part-whole relations

“quantity-mass”, “portion-object”, relating a smaller (or sub) part of an amount of matter to the whole. Two issues (glass of wine & bottle of wine vs. Salt as subquantity of SeaWater)

∀x, y(sub_quantity_of(x, y) ≡ mpart_of(x, y) ∧ M(x) ∧ M(y))

“noun-feature/activity”, entity participates in a process, like Enzyme that participates in CatalyticReaction

∀x, y(participates_in(x, y) ≡ mpart_of(x, y) ∧ ED(x) ∧ PD(y))

Using the taxonomy of part-whole relations

- Representing it correctly in ontologies and conceptual data models
  - Decision diagram
  - Using the categories of the foundational ontology
  - Examples
  - Software application that simplifies all that
- Reasoning with a taxonomy of relations
  - The RBox reasoning service to pinpoint errors
### Using the taxonomy of part-whole relations

#### Decision diagram

**Example - before**

- Envelope is not involved-in, not a member-of, does not constitute, is not a sub-quantity of, does not participate-in, is not a geographical object, but instead is contained-in the ConferenceBag.
- Transitivity holds for the mereological relations: derived facts are automatically correct, like RegistrationReceipt contained-in ConferenceBag.
- Intransitivity of Linen and ConferenceBag, because a conference bag is not wholly constituted of linen (the model does not say what the Flap is made of).
- Completeness, i.e. that all parts make up the whole, is implied thanks to the closed-world assumption. ConferenceBag directly contains the ConfProceedings and Envelope only, and does not contain, say, the Flap.
The participating objects instantiate some category (ED, PD, etc).

Given the formalization, one immediately can exclude/identify appropriate relations, taking a shortcut in the decision diagram:

- E.g.: Chewing and Eating are both a kind of (a subtype of) PD, hence involved_in
- E.g.: Alcohol and Wine are both mass nouns, or M, hence sub_quantity_of

Demo of OntoParts

Requirements for reasoning over the hierarchy:

- Represent at least Ground Mereology,
- Express ontological categories and their taxonomic relations,
- Having the option to represent transitive and intransitive relations, and
- Specify the domain and range restrictions (/relata/entity types) for the classes participating in a relation.
### The RBox Compatibility service – definitions

**Definition (Domain and Range Concepts)**

Let $R$ be a role and $R \subseteq C_1 \times C_2$ its associated Domain & Range axiom. Then, with the symbol $D_R$ we indicate the *User-defined Domain of $R$*—i.e., $D_R = C_1$—while with the symbol $R_R$ we indicate the *User-defined Range of $R$*—i.e., $R_R = C_2$.

**Definition (RBox Compatibility)**

For each pair of roles, $R, S$, such that $⟨T, R⟩ ⊑ R \subseteq S$, check:

1. $⟨T, R⟩ \not\subseteq D_S ⊓ R_S$
2. $⟨T, R⟩ \not\subseteq D_R ⊓ R_R$
3. $⟨T, R⟩ \not\subseteq R_R ⊓ R_S$

An RBox is said to be compatible iff Test 1 and (2 or 3) hold for all pairs of role-subrole in the RBox.

### The RBox Compatibility service – behaviour

- **Test 1**: $⟨T, R⟩ \not\subseteq D_R ⊓ R_R$
  - To change the role hierarchy or
  - Change domain & range restrictions
  - If the test on the domains fails, then propose a new axiom $R \subseteq D_R' \times R_R$, where $D_R' \equiv D_R \cap D_S$\(^2\), which subsequently has to go through the RBox compatibility service (and similarly when Test 1 fails on range restrictions).

- **Test 2** and **Test 3** fail: warn that $R$ cannot be a proper subrole of $S$ but that the two roles can be equivalent. Then, either:
  - Accept the possible equivalence between the two roles or
  - Change domain & range restrictions.

- Ignoring all warnings is allowed, too

---

\(^2\)The axiom $C_1 \equiv C_2$ is a shortcut for the axioms: $C_1 \subseteq C_2$ and $C_2 \subseteq C_1$. 

Extensions in various directions

- Mereotopology, with location, GIS, Region Connection Calculus (http://www.comp.leeds.ac.uk/qsr/rcc.html)
- Mereogeometry
- Mereology and vs granularity
- Temporal aspects of part-whole relations

Knowledge and Google & AfriGIS

- How can we represent
  - The Kruger Park overlaps with South Africa
  - Durban is a tangential proper part of South Africa
  - Gauteng is a non-tangential proper part of South Africa
  - Botswana is connected to South Africa (do they share a border?)
  - Lesotho is spatially located within the area of South Africa (but not part of)?
- Can we do all that with mereology? Use only spatial relations? Combining mereo+spatial?

Mereology with spatial notions

- Another primitive: Connected, which is reflexive and symmetric
- More and more expressive theories, e.g.:
  - T: \( C(x, x) \) and \( C(x, y) \rightarrow C(y, x) \)
  - MT: \( T \) and \( P(x, y) \rightarrow E(x, y) \) where \( E \) is enclosure
    \( E(x, y) = \text{def} \ \forall z \ (C(z, x) \rightarrow C(z, y)) \)
- Two primitives, \( P \) and \( C \), or part in terms of \( C \)?
  - \( P = \text{def} \ \forall z \ (C(z, x) \rightarrow C(z, y)) \)
  - or perhaps “\( x \) and \( y \) are connected parts of \( z \)” as primitive,
    \( CP(x, y, z) \), then:
    \( P(x, y) = \text{def} \ \exists z \ CP(x, z, y) \) and
    \( C(x, y) = \text{def} \ \exists z \ CP(x, y, z) \)
Some of the mereo- and topological theories

![Diagram of mereo- and mereotopological theories]

\[ \forall x, y \ (ECI(x, y) \equiv CI(x, y) \land P(y, x)) \] (18)

\[ \forall x, y \ (PCI(x, y) \equiv PPO(x, y) \land R(x) \land R(y) \land \exists z, w(has\_3D(z, x) \land has\_3D(w, y) \land ED(z) \land ED(w))) \] (19)

\[ \forall x, y \ (NTPCI(x, y) \equiv PCI(x, y) \land \forall z(C(z, x) \rightarrow O(z, y))) \] (20)

\[ \forall x, y \ (TPCI(x, y) \equiv PCI(x, y) \land \neg NTPCI(x, y)) \] (21)

\[ \forall x, y \ (ELI(x, y) \equiv LI(x, y) \land P(y, x)) \] (22)

\[ \forall x, y \ (PLI(x, y) \equiv PPO(x, y) \land R(x) \land R(y) \land \exists z, w(has\_2D(z, x) \land has\_2D(w, y) \land ED(z) \land ED(w))) \] (23)

\[ \forall x, y \ (NTPLI(x, y) \equiv PLI(x, y) \land \forall z(C(z, x) \rightarrow O(z, y))) \] (24)

\[ \forall x, y \ (TPLI(x, y) \equiv PLI(x, y) \land \neg NTPLI(x, y)) \] (25)

Extension to the taxonomy of part-whole relations

- KGEMT requires second order logic
- No definitions of relations in OWL
- Recollect object property characteristics in the different OWL species
- What do we lose exactly regarding representation and, consequently, reasoning?
- See ESWC’12 paper
Rationale

- It is hard to reuse only the “useful pieces” of a comprehensive (foundational) ontology, and the cost of reuse may be higher than developing a new ontology from scratch
- Need for small (or cleverly modularized) ontologies with explicit documentation of design rationales, and best reengineering practices
- Hence, in analogy to software design patterns: **ontology design patterns**
- ODPs summarize the good practices to be applied within design solutions
- ODPs keep track of the design rationales that have motivated their adoption

*content of slides based on Presutti et al, 2008*

ODP definition

- An ODP is an information object
- A design pattern schema is the description of an ODP, including the roles, tasks, and parameters needed in order to solve an ontology design issue
- **An ODP is a modeling solution to solve a recurrent ontology design problem. It is an Information Object that expresses a Design Pattern Schema (or skin) that can only be satisfied by DesignSolutions. Design solutions provide the setting for Ontology Elements that play some ElementRole(s) from the schema.** (Presutti et al, 2008)

ODP types

- Six families of ODPs: Structural OPs, Correspondence OPs, Content OPs (CPs), Reasoning OPs, Presentation OPs, and Lexico-Syntactic OPs
- CPs can be distinguished in terms of the domain they represent
- Correspondence OPs (for reengineering and mappings—next lecture)
- Reasoning OPs are typical reasoning procedures
- Presentation OPs relate to ontology usability from a user perspective; e.g., we distinguish between Naming OPs and Annotation OPs
- Lexico-Syntactic OP are linguistic structures or schemas that permit to generalize and extract some conclusions about the meaning they express
Logical OPs:
- Are compositions of logical constructs that solve a problem of expressivity in OWL-DL (and, in cases, also in OWL 2 DL)
- Only expressed in terms of a logical vocabulary, because their signature (the set of predicate names, e.g. the set of classes and properties in an OWL ontology) is empty
- Independent from a specific domain of interest
- Logical macros compose OWL DL constructs; e.g. the universal+existential OWL macro
- Transformation patterns translate a logical expression from a logical language into another; e.g. n-aries

Architectural OPs
Architectural OPs are defined in terms of composition of Logical OPs that are used in order to affect the overall shape of the ontology; i.e., an Architectural OP identifies a composition of Logical OPs that are to be exclusively used in the design of an ontology
Examples of Architectural OPs are: Taxonomy, Modular Architecture, and Lightweight Ontology
E.g., Modular Architecture Architectural OP consists of an ontology network, where the involved ontologies play the role of modules, which are connected by the owl:import operation with one root ontology that imports all the modules

Lexico-Syntactic OPs
- linguistic structures or schemas that consist of certain types of words following a specific order and that permit to generalize and extract some conclusions about the meaning they express; verbalisation patterns
- E.g., “subClassOf” relation, NP<subclass> be NP<superclass>, a Noun Phrase should appear before the verb—represented by its basic form or lemma, be in this example—and the verb should in its turn be followed by another Noun Phrase
- Other Lexical OPs provided for OWL’s equivalence between classes, object property, subpropertyOf relation, datatype property, existential restriction, universal restriction, disjointness, union of classes
- Mainly for English language only, thus far
- Similar to idea of ORM’s verbalization templates
Summary

1. Parts, mereology, meronymy
   - Introduction
   - Mereology
   - Implementation
   - Meronymy

2. Taxonomy of types of part-whole relations
   - The taxonomy
   - Using the taxonomy of part-whole relations
   - RBox Compatibility

3. Extensions

4. Ontology Design Patterns