COMP718: Ontologies and Knowledge Bases

Lecture 2: FOL Recap and Description Logics

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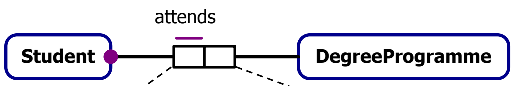
Outline

From data to ORM2 or text and then to FOL—or v.v.

Student **is an entity type**.
DegreeProgramme **is an entity type**.
Student attends DegreeProgramme.

Each Student attends **exactly one** DegreeProgramme.
It is possible that more than one Student attends **the same** DegreeProgramme.
OR, in the negative:
For each Student, **it is impossible that that** Student attends **more than one** DegreeProgramme.
It is impossible that any Student attends **no** DegreeProgramme.



Attends

Student	DegreeProgramme
John	Computer Science
Mary	Design
Fabio	Design
Claudio	Computer Science
Markus	Biology
Inge	Computer Science

How to formalise it?

- Note: logic is not the study of truth, but of the relationship between the truth of one statement and that of another
- Syntax
  - Alphabet
  - Languages constructs
  - Sentences to assert knowledge
- Semantics
  - Formal meaning

First order logic

The lexicon of a first order language contains:

- Connectives & Parentheses: $\neg$, $\rightarrow$, $\leftrightarrow$, $\wedge$, $\vee$, (and);
- Quantifiers: $\forall$ (universal) and $\exists$ (existential);
- Variables: $x, y, z, \ldots$ ranging over particulars;
- Constants: $a, b, c, \ldots$ representing a specific element;
- Functions: $f, g, h, \ldots$, with arguments listed as $f(x_1, \ldots x_n)$;
- Relations: $R, S, \ldots$ with an associated arity.

Example: From Natural Language to First order logic (or vv.)

- Each animal is an organism
  All animals are organisms
  If it is an animal then it is an organism
  $\forall x(Animal(x) \rightarrow Organism(x))$
- Each student must be registered for a degree programme
  $\forall x(registered\_for(x, y) \rightarrow Student(x) \wedge DegreeProgramme(y))$
  $\forall x(Student(x) \rightarrow \exists y \ registered\_for(x, y))$
- Aliens exist
  $\exists x \ Alien(x)$
- There are books that are heavy
  $\exists x(Book(x) \wedge heavy(x))$

First order logic

- (countably infinite) Supply of **symbols** (signature): Variables, Functions , Constants, and Relations
- **Terms**: A term is inductively defined by two rules, being:
  1. Every variable and constant is a term.
  2. if f is a m-ary function and $t_1, \ldots t_m$ are terms, then $f(t_1, \ldots, t_m)$ is also a term.

Definition (atomic formula)

An *atomic formula* is a formula that has the form $t_1 = t_2$ or $R(t_1, \ldots, t_n)$ where R is an n-ary relation and $t_1, \ldots, t_n$ are terms.

R1. If ϕ is a formula then so is $\neg\phi$.
R2. If ϕ and ψ are formulas then so is $\phi \wedge \psi$.
R3. If ϕ is a formula then so is $\exists x \phi$ for any variable x.

FOL Cont.

Definition (formula)

A string of symbols is a *formula* of FOL if and only if it is constructed from atomic formulas by repeated applications of rules R1, R2, and R3.

A *free variable* of a formula ϕ is that variable occurring in ϕ that is not quantified. We then can introduce the definition of *sentence*.

Definition (sentence)

A *sentence* of FOL is a formula having no free variables.

FOL Cont.: toward semantics

- Whether a sentence is true or not depends on the underlying set and the interpretation of the function, constant, and relation symbols.
- A *structure* consists of an *underlying set* together with an *interpretation* of functions, constants, and relations.
- Given a sentence ϕ and a structure M, M *models* ϕ means that the sentence ϕ is true with respect to M. More precisely,

Definition (vocabulary)

A *vocabulary* $\mathcal{V}$ is a set of function, relation, and constant symbols.

FOL Cont.

Definition ($\mathcal{V}$-structure)

A $\mathcal{V}$-*structure* consists of a non-empty underlying set Δ along with an interpretation of $\mathcal{V}$. An interpretation of $\mathcal{V}$ assigns an element of Δ to each constant in $\mathcal{V}$, a function from Δ^n to Δ to each n-ary function in $\mathcal{V}$, and a subset of Δ^n to each n-ary relation in $\mathcal{V}$. We say M is a *structure* if it is a $\mathcal{V}$-structure of some vocabulary $\mathcal{V}$.

Definition ($\mathcal{V}$-formula)

Let $\mathcal{V}$ be a vocabulary. A $\mathcal{V}$-*formula* is a formula in which every function, relation, and constant is in $\mathcal{V}$. A $\mathcal{V}$-*sentence* is a $\mathcal{V}$-formula that is a sentence.

FOL Cont.

- When we say that M *models* ϕ, denoted with $M \models \phi$, this is with respect to M being a $\mathcal{V}$-structure and $\mathcal{V}$-sentence ϕ is true in M.
- Model theory: the interplay between M and a set of first-order sentences $\mathcal{T}(M)$, which is called the *theory of* M, and its 'inverse' from a set of sentences Γ to a class of structures.

Definition (theory of M)

For any $\mathcal{V}$-structure M, the *theory of* M, denoted with $\mathcal{T}(M)$, is the set of all $\mathcal{V}$-sentences ϕ such that $M \models \phi$.

Definition (model)

For any set of $\mathcal{V}$-sentences, a *model* of Γ is a $\mathcal{V}$-structure that models each sentence in Γ. The class of all models of Γ is denoted by $\mathcal{M}(\Gamma)$.

Theory in the context of logic

Definition (complete $\mathcal{V}$-theory)

Let Γ be a set of $\mathcal{V}$-sentences. Then Γ is a *complete* $\mathcal{V}$-*theory* if, for any $\mathcal{V}$-sentence ϕ either ϕ or $\neg\phi$ is in Γ and it is not the case that both ϕ and $\neg\phi$ are in Γ.

- It can then be shown that for any $\mathcal{V}$-structure M, $\mathcal{T}(M)$ is a complete $\mathcal{V}$-theory (for proof, see *e.g.* [Hedman04, p90])

Definition

A set of sentences Γ is said to be *consistent* if no contradiction can be derived from Γ.

Definition (theory)

A *theory* is a consistent set of sentences.

Some definitions

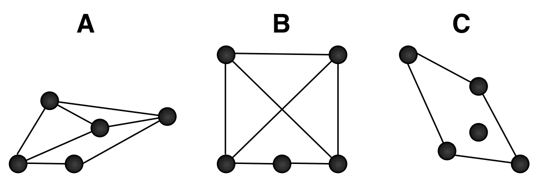
- A formula is valid if it holds under *every* assignment. $\models \phi$ to denote this. A valid formula is called a tautology.
- A formula is satisfiable if it holds under *some* assignment.
- A formula is unsatisfiable if it holds under *no* assignment. An unsatisafiable formula is called a contradiction.

Example

- Is this a theory?
  $\forall x(Woman(x) \rightarrow Female(x))$
  $\forall x(Mother(x) \rightarrow Woman(x))$
  $\forall x(Man(x) \leftrightarrow \neg Woman(x))$
  $\forall x(Mother(x) \rightarrow \exists y(partnerOf(x, y) \wedge Spouse(y))$
  $\forall x(Spouse(x) \rightarrow Man(x) \vee Woman(x))$
  $\forall x, y(Mother(x) \wedge partnerOf(x, y) \rightarrow Father(y))$
- Is it still a theory if we add:
  $\forall x(Hermaphrodite(x) \rightarrow Man(x) \wedge Woman(x))$

Examples of first-order structures (exercise)

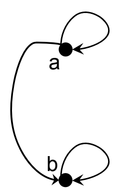
- Graphs are mathematical structures.
- A graph is a set of points, called **vertices**, and lines, called **edges** between them. For instance:



- Figures A and B are different depictions, but have the same descriptions w.r.t. the vertices and edges. Check this.
- Graph C has a property that A and B do not have. Represent this in a first-order sentence.
- Find a suitable first-order language for A (/B), and formulate at least two properties of the graph using quantifiers.

More graphs (exercise)

Consider the following graph, and first-order language $\mathcal{L} = \langle R \rangle$, with R being a binary relation symbol (edge).



1. Formalise the following properties of the graph as $\mathcal{L}$-sentences: (i) (a, a) and (b, b) are edges of the graph; (ii) (a, b) is an edge of the graph; (iii) (b, a) is not an edge of the graph. Let T stand for the resulting set of sentences.
2. Prove that $T \cup \{\forall x \forall y R(x, y)\}$ is unsatisfiable using tableaux calculus.

Reasoning

- Representing the knowledge in a suitable logic is one thing, reasoning over it another. e.g.:
  - How do we find out whether a formula is valid or not?
  - How do we find out whether our knowledge base is satisfiable?
- Among others:
  - Truth tables
  - Tableaux (principal approach for DL reasoners)
  - ...

Tableaux summary (1/2)

- A sound and complete procedure deciding satisfiability is all we need, and the **tableaux method is a decision procedure which checks the existence of a model**
- It exhaustively looks at all the possibilities, so that it can eventually prove that no model could be found for unsatisfiable formulas.
- $\phi \models \psi$ iff $\phi \land \neg\psi$ is NOT satisfiable—if it is satisfiable, we have found a counterexample
- Decompose the formula in top-down fashion

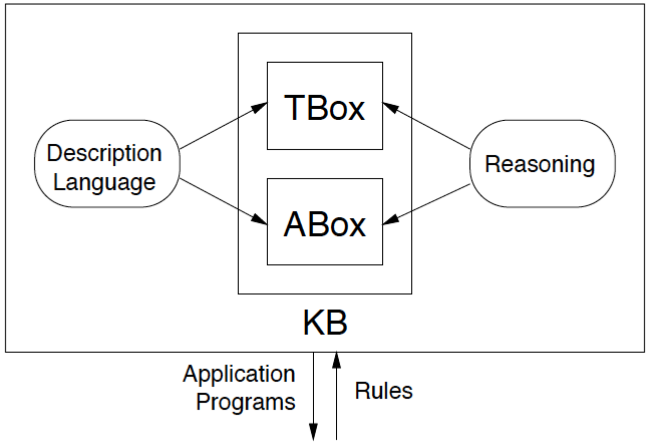
Tableaux summary (2/2)

- Tableaux calculus works only if the formula has been translated into **Negation Normal Form**, *i.e.*, all the negations have been pushed inside
- Recollect the list of equivalences, apply those to arrive at NNF, if necessary.
- If a model satisfies a conjunction, then it also satisfies each of the conjuncts
- If a model satisfies a disjunction, then it also satisfies one of the disjuncts. It is a non-deterministic rule, and it generates two alternative branches.
- Apply the completion rules until either (a) an explicit contradiction due to the presence of two opposite literals in a node (a clash) is generated in each branch, or (b) there is a completed branch where no more rule is applicable.

What are DLs?

- A logical reconstruction and (claimed to be a) unifying formalism for other KR languages, such as frames-based systems, OO modelling, Semantic data models, etc.
- A structured fragment of FOL
- Representation is at the predicate level: no variables are present in the formalism
- Any (basic) Description Logic is a subset of $\mathcal{L}_3$, i.e., the function-free FOL using only at most three variable names.
- Provide theories and systems for declaratively expressing structured information and for accessing and reasoning with it.
- Used for, a.o.: terminologies and ontologies, formal conceptual data modelling, information integration, ...

Description Logic knowledge base



$\mathcal{ALC}$

- Concepts denoting entity types/classes/unary predicates/universals, including top $\top$ and bottom $\bot$;
- Roles denoting relationships/associations/n-ary predicates/properties;
- Constructors: and $\sqcap$, or $\sqcup$, and not $\neg$; quantifications forall $\forall$ and exists $\exists$
- Complex concepts using constructors
  - Let C and D be concept names, R a role name, then
  - $\neg C$, $C \sqcap D$, and $C \sqcup D$ are concepts, and
  - $\forall R.C$ and $\exists R.C$ are concepts
- Individuals

$\mathcal{ALC}$ Examples

- Concepts (primitive, atomic): $\mathtt{Book}$, $\mathtt{Course}$
- Roles: $\mathtt{ENROLLED}$, $\mathtt{READS}$
- Complex concepts:
  - $\mathtt{Student} \sqsubseteq \exists\mathtt{ENROLLED}.(\mathtt{Course} \sqcup \mathtt{DegreeProgramme})$ (a *primitive concept*)
  - $\mathtt{Mother} \sqsubseteq \mathtt{Woman} \sqcap \exists\mathtt{PARENTOF}.\mathtt{Person}$
  - $\mathtt{Parent} \equiv (\mathtt{Male} \sqcup \mathtt{Female}) \sqcap \exists\mathtt{PARENTOF}.\mathtt{Mammal} \sqcap \exists\mathtt{CARESFOR}.\mathtt{Mammal}$ (a *defined concept*)
- Individuals: $\mathtt{Student(Shereen)}$, $\mathtt{Mother(Sally)}$, $\neg\mathtt{Student(Sally)}$, $\mathtt{ENROLLED(Shereen, COMP101)}$
- But what does it really mean?

Semantics of $\mathcal{ALC}$ (1/3)

- Domain Δ is a non-empty set of objects
- Interpretation: $\cdot^{\mathcal{I}}$ is the *interpretation function*, domain $\Delta^{\mathcal{I}}$
  - $\cdot^{\mathcal{I}}$ maps every concept name A to a subset $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - $\cdot^{\mathcal{I}}$ maps every role name R to a subset $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
  - $\cdot^{\mathcal{I}}$ maps every individual name a to elements of $\Delta^{\mathcal{I}}$: $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- Note: $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$ and $\bot^{\mathcal{I}} = \emptyset$

Semantics of $\mathcal{ALC}$ (2/3)

- C and D are concepts, R a role
- $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \backslash C^{\mathcal{I}}$
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $(\forall R.C)^{\mathcal{I}} = \{x \mid \forall y.R^{\mathcal{I}}(x,y) \rightarrow C^{\mathcal{I}}(y)\}$
- $(\exists R.C)^{\mathcal{I}} = \{x \mid \exists y.R^{\mathcal{I}}(x,y) \wedge C^{\mathcal{I}}(y)\}$

Semantics of $\mathcal{ALC}$ (3/3)

- C and D are concepts, R a role, a and b are individuals
- An interpretation $\mathcal{I}$ satisfies the statement $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- An interpretation $\mathcal{I}$ satisfies the statement $C \equiv D$ if $C^{\mathcal{I}} = D^{\mathcal{I}}$
- $C(a)$ is satisfied by $\mathcal{I}$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
- $R(a,b)$ is satisfied by $\mathcal{I}$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

- An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is a model of a knowledge base $\mathcal{KB}$ if every axiom of $\mathcal{KB}$ is satisfied by $\mathcal{I}$
- A knowledge base $\mathcal{KB}$ is said to be satisfiable if it admits a model

Logical implication

- $\mathcal{KB} \models \phi$ if every model of $\mathcal{KB}$ is a model of ϕ
- Example:
  TBox: $\exists\texttt{TEACHES.Course} \sqsubseteq \neg\texttt{Undergrad} \sqcup \texttt{Professor}$
  ABox: $\texttt{TEACHES(John, cs101)}$, $\texttt{Course(cs101)}$, $\texttt{Undergrad(John)}$
- $\mathcal{KB} \models \texttt{Professor(John)}$
- What if:
  TBox: $\exists\texttt{TEACHES.Course} \sqsubseteq \texttt{Undergrad} \sqcup \texttt{Professor}$
  ABox: $\texttt{TEACHES(John, cs101)}$, $\texttt{Course(cs101)}$, $\texttt{Undergrad(John)}$
- $\mathcal{KB} \models \texttt{Professor(John)}$? or perhaps $\mathcal{KB} \models \neg\texttt{Professor(John)}$?

Reasoning services for DL-based OWL ontologies

- Consistency of the knowledge base ($\mathcal{KB} \not\models \top \sqsubseteq \bot$)
  - Is the $\mathcal{KB} = (\mathcal{T}, \mathcal{A})$ consistent (non-selfcontradictory), i.e., is there at least a model for $\mathcal{KB}$?
- Concept (and role) satisfiability ($\mathcal{KB} \not\models C \sqsubseteq \bot$)
  - is there a model of $\mathcal{KB}$ in which C (resp. R) has a nonempty extension?
- Concept (and role) subsumption ($\mathcal{KB} \models C \sqsubseteq D$)
  - i.e., is the extension of C (resp. R) contained in the extension of D (resp. S) in every model of $\mathcal{T}$?
- Instance checking ($\mathcal{KB} \models C(a)$ or $\mathcal{KB} \models R(a,b)$)
  - is a (resp. (a,b)) a member of concept C (resp. R) in $\mathcal{KB}$, i.e., is the fact $C(a)$ (resp. $R(a,b)$) satisfied by every interpretation of $\mathcal{KB}$?
- Instance retrieval ($\{a \mid \mathcal{KB} \models C(a)\}$)
  - find all members of C in $\mathcal{KB}$, i.e., compute all individuals a s.t. $C(a)$ is satisfied by every interpretation of $\mathcal{KB}$

Tableau reasoning

- Most common for DL reasoners
- Like for FOL:
  - Unfold the TBox
  - Convert the result into negation normal form
  - Apply the tableau rules to generate more Aboxes
  - Stop when none of the rules are applicable
- $\mathcal{T} \vdash C \sqsubseteq D$ if all Aboxes contain clashes
- $\mathcal{T} \nvdash C \sqsubseteq D$ if some Abox does not contain a clash

Negation Normal Form

- C and D are concepts, R a role
- $\neg$ only in front of concepts:
  - $\neg\neg C$ gives C
  - $\neg(C \sqcap D)$ gives $\neg C \sqcup \neg D$
  - $\neg(C \sqcup D)$ gives $\neg C \sqcap \neg D$
  - $\neg(\forall R.C)$ gives $\exists R.\neg C$
  - $\neg(\exists R.C)$ gives $\forall R.\neg C$

Tableau rules

$\sqcap$-rule If $(C_1 \sqcap C_2)(a) \in S$ but S does not contain both $C_1(a)$ and $C_2(a)$, then
$S = S \cup \{C_1(a), C_2(a)\}$

$\sqcup$-rule If $(C_1 \sqcup C_2)(a) \in S$ but S contains neither $C_1(a)$ nor $C_2(a)$, then
$S = S \cup \{C_1(a)\}$
$S = S \cup \{C_2(a)\}$

$\forall$-rule If $(\forall R.C)(a) \in S$ and S contains $R(a, b)$ but not $C(b)$, then
$S = S \cup \{C(b)\}$

$\exists$-rule If $(\exists R.C)(a) \in S$ and there is no b such that $C(b)$ and $R(a, b)$, then
$S = S \cup \{C(b), R(a, b)\}$

Summary

1. FOL Recap
   - Syntax
   - Semantics
   - First Order Structures
2. Description logics
   - Introduction
   - Basic DL: $\mathcal{ALC}$
   - Reasoning services