COMP718: Ontologies and Knowledge-based Systems
Answers to Exercises Chapter 2

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1 FOL

1. Is the following argument valid, a tautology, a contradiction, satisfiable, or neither? Represent the argument formally in propositional logic and use truth tables to prove it.
   Dalila travels to Johannesburg or she travels to Durban.
   If she travels to Johannesburg, she takes the plane.
   Therefore, Dalila does not travel to Durban.
   [**Answer**]
   A: Dalila travels to Johannesburg
   B: Dalila travels to Durban
   C: Dalila takes the plane
   Then we can formalise the three sentences, above as: $((A \lor B) \land (A \to C)) \to \neg B$
   Truth table: satisfiable.

```
A   B C (((A v B) ^ (A => C)) => ~ B)
T   T T F
T   T F T
T   F T T
T   F F T
F   T T F
F   T F F
F   F T T
F   F F T
```

2. Consider the structures in Figure 1, which are graphs.
   (a) Figures 1-A and B are different depictions, but have the same descriptions w.r.t. the vertices and edges. Check this.
   (b) C has a property that A and B do not have. Represent this in a first-order sentence.
   [**Answer**]
   There exists a node that does not participate in an instance of R, or: it does not relate to anything else: $\exists x \forall y. \neg R(x, y)$.
   (c) Find a suitable first-order language for A (/B), and formulate at least two properties of the graph using quantifiers.
   [**Answer**]
   $\mathcal{L} = \langle R \rangle$ as the binary relation between the vertices. Optionally, on can add the vertices as well. Properties:
   R is symmetric: $\forall xy. R(x, y) \to R(y, x)$.
   R is irreflexive: $\forall x. \neg R(x, x)$.

If you take into account the vertices explicitly, one could say that each note participates in at least two instances of R to different nodes.

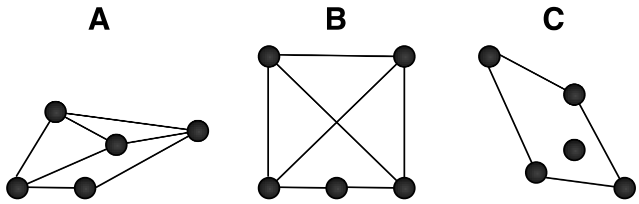

Figure 1: Graphs for Question 2.

3. Consider the graph in Figure 2, and first-order language $\mathcal{L} = \langle R \rangle$, with R being a binary relation symbol (edge).
   (a) Formalise the following properties of the graph as $\mathcal{L}$-sentences: (i) (a, a) and (b, b) are edges of the graph; (ii) (a, b) is an edge of the graph; (iii) (b, a) is not an edge of the graph. Let T stand for the resulting set of sentences.
   [**Answer**]
   R is reflexive (a thing relates to itself): $\forall x.R(x, x)$.
   R is asymmetric (if a relates to b through relation R, then b does not relate back to a through R): $\forall xy.R(x, y) \rightarrow \neg R(y, x)$.
   (b) Prove that $T \cup \{\forall x \forall y R(x, y)\}$ is unsatisfiable using tableaux calculus.
   [**Answer**]
   See the example on p21 of the lecture notes.

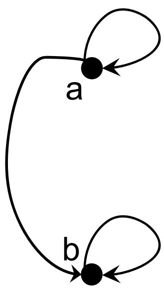

Figure 2: Graph for Question 3.

2 Description Logics

1. Explain in your own words what the following $\mathcal{ALC}$ reasoning tasks involve and why they are important for reasoning with ontologies:
   a. Instance checking.
   b. Subsumption checking.
   c. Checking for concept satisability.

2. Consider the following TBox $\mathcal{T}$:
   $Vegan \equiv Person \sqcap \forall eats.Plant$
   $Vegetarian \equiv Person \sqcap \forall eats.(Plant \sqcup Dairy)$
   We want to know if $\mathcal{T} \vdash Vegan \sqsubseteq Vegetarian$.
   This we convert to a constraint system $S = \{(Vegan \sqcap \neg Vegetarian)(a)\}$,

which is unfolded (here: complex concepts on the left-hand side are replaced with their properties declared on the right-hand side) into:

$$S = \{Person \sqcap \forall eats.Plant \sqcap \neg(Person \sqcap \forall eats.(Plant \sqcup Dairy))(a)\} \tag{1}$$

Tasks:

a. Rewrite (Eq. 1) into negation normal form
   [Answer]
   $Person \sqcap \forall eats.Plant \sqcap (\neg Person \sqcup \neg\forall eats.(Plant \sqcup Dairy))$
   $Person \sqcap \forall eats.Plant \sqcap (\neg Person \sqcup \exists\neg eats.(Plant \sqcup Dairy))$
   $Person \sqcap \forall eats.Plant \sqcap (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))$
   So our initial ABox is:
   $S = \{(Person \sqcap \forall eats.Plant \sqcap (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy)))(a)\}$
b. Enter the tableau by applying the rules (see lecture slide 31) until either you find a completion or only clashes.
   [Answer]
   ($\sqcap$-rule): $\{Person(a), \forall eats.Plant(a), (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))(a)\}$
   ($\sqcup$-rule):
   (1) $\{Person(a), \forall eats.Plant(a), (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))(a), \neg Person(a)\}$
   ¡clash!
   (2) $\{Person(a), \forall eats.Plant(a), (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))(a), \exists eats.(\neg Plant \sqcap \neg Dairy)(a)\}$
   ($\exists$-rule): $\{Person(a), \forall eats.Plant(a), (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))(a), \exists eats.(\neg Plant \sqcap \neg Dairy)(a), eats(a,b), (\neg Plant \sqcap \neg Dairy)(b)\}$
   ($\sqcap$-rule): $\{Person(a), \forall eats.Plant(a), (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))(a), \exists eats.(\neg Plant \sqcap \neg Dairy)(a), eats(a,b), (\neg Plant \sqcap \neg Dairy)(b), \neg Plant(b), \neg Dairy(b)\}$
   ($\forall$-rule): $\{Person(a), \forall eats.Plant(a), (\neg Person \sqcup \exists eats.(\neg Plant \sqcap \neg Dairy))(a), \exists eats.(\neg Plant \sqcap \neg Dairy)(a), eats(a,b), (\neg Plant \sqcap \neg Dairy)(b), \neg Plant(b), \neg Dairy(b), Plant(b)\}$ ¡clash!
c. $\mathcal{T} \vdash Vegan \sqsubseteq Vegetarian$?
   [Answer]
   yes