Semantic Web Technologies

Lecture 7: Ontology engineering: uncertainty and vagueness

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Outline

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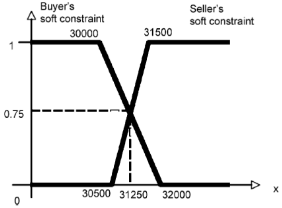
Examples

- Information Retrieval: To which degree is a Web site, a Web page, a text passage, an image region, a video segment, . . . relevant to my information need?[1]
- Matchmaking: To which degree does an object match my requirements? e.g., your budget is about 20.000 euro to buy a car, then to which degree does a cars price of 20.500 euro match your budget?
- Ontology alignment: To which degree do two concepts of two ontologies represent the same thing, or are disjoint, or are overlapping?
- Classifying ripe apples or "the set of all individuals that mostly buy low calorie food"

[1] some of the following slides are taken from Umberto Straccia's AAAI'07 tutorial [http://gaia.isti.cnr.it/~straccia/download/papers/VANCOUVER07/VANCOUVER07.pdf]

- A car seller sells an Audi TT for 31500 euro (catalog price)
- A buyer is looking for a sports-car, but wants to to pay not more than around 30000 euro
- Classical DLs: the problem relies on the crisp conditions on price
- More fine grained approach (as usual in negotiation): consider prices as vague constraints (fuzzy sets)
    - Seller would sell above 31500 euro, but can go down to 30500
    - The buyer prefers to spend less than 30000 euro, but can go up to 32000 euro
    - Highest degree of matching is 0.75; The car may be sold at 31250 euro



- Problems: what and how to incorporate such vague or uncertain knowledge in OWL and its reasoners?
- Solutions:
  i. probabilistic, possibilistic, fuzzy, rough extensions to the language
  ii. for reasoning: transform back into OWL and use standard reasoner or develop your own one
- Usage, among others:
  - Information retrieval (e.g., top-k retrieval)
  - classifying patients (e.g., patients that are possibly septic have properties: infection and [temperature > 38C OR temperature < 36C, respiratory rate > 20 breaths/minute OR PaCO2 < 32 mmHg, etc])
  - Recommender systems (user preferences etc.)
  - Matchmaking in web services

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Uncertainty and vagueness

- **Uncertainty**: statements are true or false, but due to lack of knowledge we can only estimate to which probability / possibility / necessity degree they are true or false
  - E.g.: a bird flies or does not fly. The probability / possibility / necessity degree that it flies is 0.83
- **Vagueness**: statements involve concepts for which there is no exact definition, such as tall, small, close, far, cheap, expensive. true to some degree, taken from a truth space
  - E.g., "Hotel Verdi is close to the train station to degree 0.83"
- Uncertainty *and* Vagueness: "It is *probable* to degree 0.83 that it will be *hot* tomorrow"
- Imperfect information covers notions such as uncertainty, vagueness, contradiction, incompleteness, imprecision

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- Finite nonempty set of basic events $\Phi = \{p_1, \ldots, p_n\}$, with $n \geq 1$
- *Events*: every element of $\Phi \cup \{\bot, \top\}$ is an event; if ϕ and ψ are events, then so are $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, and $(\phi \rightarrow \psi)$
- A *probabilistic formula* is an expression of the form $\phi \geq l$, with $l \in \mathbb{R}$ from the unit interval $[0, 1]$ (note that $\neg\phi \geq 1 - u$ encodes ϕ is true with probability at most u)
- *Conditional constraint* $(\psi \mid \phi)[l, u]$: events ψ and ϕ, and $l, u \in [0, 1]$, which denotes "the conditional probability of ψ given ϕ is in $[l, u]$"
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Probabilistic logic: Semantics

- A world I associates with every basic event in Φ a binary truth value, and extend I by induction to all events as usual
- $\mathcal{I}_\Phi$ is the (finite) set of all worlds for Φ
- A world I satisfies an event ϕ (or: I is a model of ϕ), denoted $I \models \phi$, iff $I(\phi) = \mathit{true}$
- Probabilistic interpretation Pr: probability function on $\mathcal{I}_\Phi$ s.t. all $Pr(I)$ with $I \in \mathcal{I}_\Phi$ sum up to 1
- $Pr(\phi)$ is the sum of all $Pr(I)$ such that $I \in \mathcal{I}_\Phi$ and $I \models \phi$
- $Pr(\psi \mid \phi)$: if $Pr(\phi) > 0$, then $Pr(\psi \mid \phi) = \frac{Pr(\psi \wedge \phi)}{Pr(\phi)}$

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Probabilistic logic: satisfiability and entailment

- A probabilistic interpretation Pr *satisfies* a probabilistic formula $\phi \geq l$ (i.e., $Pr \models \phi \geq l$) iff $Pr(\phi) \geq l$
- Pr satisfies a probabilistic KB iff Pr satisfies all $F \in KB$
- KB is satisfiable iff a model of KB exists
- A probabilistic formula F is a *logical consequence* of KB (denoted $KB \models F$) iff every model of KB satisfies F
- $\phi \geq l$ is a *tight logical consequence* of KB iff l is the infimum[2] of $Pr(\phi)$ subject to all models Pr of KB (the latter is equivalent to $l = sup\{r \mid KB \models \phi \geq r\}$)[3]

[2] the infimum of a subset of some set is the greatest element (not necessarily in the subset) that is less than or equal to all elements of the subset; greatest lower bound.

[3] the supremum (sup) of a subset S of a partially ordered set T is the least element of T that is greater than or equal to each element of S; least upper bound.

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Probabilistic RDF, OWL, and DLs

- P-$\mathcal{SHOQ}(D)$, P-$\mathcal{SHOIN}(D)$ (by T. Lukasiewicz)
  - uses the notion of a conditional constraint
  - semantics is based on the notion of lexicographic entailment in probabilistic default reasoning
  - probabilistic TBox and ABox
  - interprets TBox and ABox probabilistic knowledge as statistical knowledge and as degrees of belief about instances of concepts and roles, respectively
  - allows for deriving both statistical knowledge and degrees of belief
  - allows for expressing default knowledge about concepts
- PR-OWL (by da Costa and Laskey)
  - Probabilistic semantics based on multi-entity Bayesian networks
- And others with Bayesian networks, with DLs, covering various permutations of probabilistic KR&R added to different languages (see references in Straccia, 2008)

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Use of Probabilistic Ontologies

- Representation of terminological and assertional probabilistic knowledge (e.g., in the medical domain or at the stock exchange market)
- Information retrieval, for an increased recall
- Ontology matching
- Probabilistic data integration, especially for handling ambiguous and controversial pieces of information

Outline

Possibilistic logic introduction

- Syntactically, we now use possibilistic formulas to constrain the necessities and possibilities of propositional events
- Semantically, we now have possibility distributions on worlds, each of which associates with every event a unique possibility and a unique necessity
- Differently from the probability of an event (sum of the probabilities of all worlds that satisfy that event), the possibility of an event is the *maximum of the possibilities* of all worlds that satisfy the event
- Possibilistic logic useful for encoding user preferences, since possibility measures can be viewed as rankings (on worlds or also objects) along an ordinal scale
- While reasoning in probabilistic logic generally requires to solve linear optimization problems, reasoning in possibilistic logic does not and thus can generally be done with less computational effort

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- While reasoning in probabilistic logic generally requires to solve linear optimization problems, reasoning in possibilistic logic does not and thus can generally be done with less computational effort

Possibilistic logic introduction

- Syntactically, we now use possibilistic formulas to constrain the necessities and possibilities of propositional events
- Semantically, we now have possibility distributions on worlds, each of which associates with every event a unique possibility and a unique necessity
- Differently from the probability of an event (sum of the probabilities of all worlds that satisfy that event), the possibility of an event is the *maximum of the possibilities* of all worlds that satisfy the event
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Possibilistic logic: Syntax and Semantics

- Possibilistic formulas have the form $\mathrm{P}\phi \geq l$ or $\mathrm{N}\phi \geq l$, with ϕ event, $l \in \mathbb{R}$ from [0, 1], **P**ossibly, and **N**ecessarly. e.g.:
  - $\mathrm{P}\mathit{snow\_today} \geq 0.7$ encodes that it will snow today is possible to degree 0.7
  - $\mathrm{N}\mathit{mother} \rightarrow \mathit{female} \geq 1$ says that a mother is necessarily female
- A *possibilistic formula* is a pair (ϕ, α) consisting of a classical logic formula ϕ and a degree α expressing certainty or priority (which also can be considered as possibility degree of ϕ)
- A *possibilistic knowledge base KB* is a finite set of possibilistic formulas, of the form $KB = \{(\phi_i, \alpha_i) : i = 1 \ldots n\}$
- A *possibilistic interpretation* is a mapping $\pi : \mathcal{I}_\Phi \rightarrow [0, 1]$
- $\pi(I)$ is the degree to which world I is possible
  - every world I such that $\pi(I) = 0$ is impossible
  - every world I such that $\pi(I) = 1$ is totally possible
  - π is normalized iff $\pi(I) = 1$ for some $I \in \mathcal{I}_\Phi$

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- The possibility of a ϕ in a π defined by
  $Poss(\phi) = max\{\pi(I) \mid I \in \mathcal{I}_{\Phi}, I \models \phi\}$
  'possibility of ϕ is evaluated in the most possible world where ϕ is true'
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  'to what extent ϕ is certainly true'
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- The *inconsistency degree* of KB, denoted $Inc(KB)$, is defined as $Inc(KB) = max\{\alpha_i : KB_{\geq \alpha_i} is\ inconsistent\}$
- There are two possible definitions of inference in possibilistic logic:
  - A formula ϕ is said to be a *plausible consequence* of KB, denoted by $KB \vdash_P \phi$, iff $KB_{>Inc(KB)} \vdash \phi$
  - A formula ϕ is said to be a *possibilistic consequence* of KB to degree α, denoted by $KB \vdash_\pi (\phi, \alpha)$, iff the following conditions hold: (1) $KB_{\geq\alpha}$ is consistent, (2) $KB_{\geq\alpha} \vdash \phi$, and (3) $\forall\beta > \alpha$, $KB_{\geq\beta} \nvdash \phi$
- Inference services: instance checking (plausible instance of C), plausible subsmption, instance checking with necessity degree, subsumption with necessity degree

Possibilistic ontologies

- Add it to an arbitrary DL language (including any of the DL-based OWL languages)
- E.g. Qi, Pan, Ji in DL'07, supposedly with basics implemented in KAON2
- Possibilistic generalization of $\mathcal{ALC}$ for information retrieval (Liau and Yao, 2001), used for query relaxation, restriction, and exemplar-based retrieval
- Thus far: little usage, examples are toy examples

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Outline

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Introduction

- Vagueness: statements involve concepts for which there is no exact definition, such as tall, close
- Statements are true to some degree which is taken from a *truth space*, which is usually $[0, 1]$
  - Hotel Verdi is close to the train station to degree 0.83
  - Find top-k cheapest hotels close to the train station: $q(h) \leftarrow hasLocation(h, hl) \wedge hasLocation(train, cl) \wedge close(hl, cl) \wedge cheap(h)$
  - What is the interpretation of $close(verdi, train) \wedge cheap(200)$?
  - Interpretation: a function I mapping atoms into $[0, 1]$, i.e. $I(A) \in [0, 1]$
  - if $I(close(verdi, train)) = 0.83$ and $I(cheap(200)) = 0.2$, then what is the result of $0.83 \wedge 0.2$?
- More generally, what is the result of $n \wedge m$, for $n, m \in [0, 1]$?

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Fuzzy logic (basics)

- Formulae: First-Order Logic formulae, terms are either variables or constants
- many-valued formulae have the form $\phi \geq l$ or $\phi \leq u$ where $l, u \in [0,1]$ (degree of truth is *at least* l and *at most* u, resp.)
- Formulae have a degree of truth in truth space $[0,1]$
- Interpretation is a mapping $I : Atoms \to [0,1]$, which are extended to formulae as follows (subsection):

$$\mathcal{I}(\neg\phi) = \mathcal{I}(\phi) \to 0 \quad (1)$$

$$\mathcal{I}(\exists x \phi) = \sup_{c \in \Delta^{\mathcal{I}}} \mathcal{I}_x^c(\phi) \quad (2)$$

$$\mathcal{I}(\forall x \phi) = \inf_{c \in \Delta^{\mathcal{I}}} \mathcal{I}_x^c(\phi) \quad (3)$$

$$\mathcal{I}(\phi \wedge \psi) = \mathcal{I}(\phi) \otimes \mathcal{I}(\psi) \quad (4)$$

$$\mathcal{I}(\phi \vee \psi) = \mathcal{I}(\phi) \oplus \mathcal{I}(\psi) \quad (5)$$

$$\mathcal{I}(\phi \to \psi) = \mathcal{I}(\phi) \Rightarrow \mathcal{I}(\psi) \quad (6)$$

$$\mathcal{I}(\neg\phi) = \ominus \mathcal{I}(\phi) \quad (7)$$

cont'd

- where $\mathcal{I}^c_x$ is as $\mathcal{I}$ except that var x is mapped to individual c
- $\otimes$, $\oplus$, $\Rightarrow$, and $\ominus$ are *combination functions*: triangular norms (or t-norms), triangular co-norms (or s-norms), implication functions, and negation functions, respectively
- which extend the classical Boolean conjunction, disjunction, implication, and negation, respectively, to the many-valued case
- Degree of subsumption between two fuzzy sets A and B, denoted $A \sqsubseteq B$, is defined as $\inf_{x \in X} A(x) \Rightarrow B(x)$
  - If $A(x) \leq B(x)$ for all $x \in [0,1]$ then $A \sqsubseteq B$ evaluates to 1
- $\mathcal{I} \models \phi \geq l$ (resp. $\mathcal{I} \models \phi \leq u$) iff $\mathcal{I}(\phi) \geq l$ (resp. $\mathcal{I}(\phi) \leq u$)

Fuzzy RDF and RDFS

- Fuzzy RDF
- Statement (triples) may have attached a degree in $[0, 1]$: for $n \in [0, 1]$
  - $\langle(subject, predicate, object), n\rangle$
  - Meaning: the degree of truth of the statement is at least n
  - E.g.: $\langle(o1, isAbout, snoopy), 0.8\rangle$
- Inferences, e.g.: $\frac{\langle(a,subClassOf,b),n\rangle,\langle(x,type,a),m\rangle}{\langle(x,type,b),n\wedge m\rangle}$
- Fuzzy RDFS adds extra constraints on interpretations
- see, e.g., 'A fuzzy semantics for semantic web languages' by Mazzieri and Dragoni, 2005

Fuzzy DLs can be classified according to:

- the description logic resp. ontology language that they generalize
- the allowed fuzzy constructs and the underlying fuzzy logics (Gödel, Lukasiewicz, Zadeh, ...)
- their reasoning services:
  - Consistency, Subsumption, Equivalence
  - Graded instantiation: Check if individual a is an instance of class C to degree at least n, i.e., $KB \models \langle a : C, n \rangle$
  - Best Truth Value Bound problem: determine tightest bound $n \in [0, 1]$ of an axiom α, i.e. $glb(KB, \alpha) = \sup\{n \mid KB \models (\alpha \geq n)\}$ (likewise for lub)
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  - $glb(KB, C \sqsubseteq D)$ is the minimal value of x such that $KB = (\mathcal{R}, \mathcal{T}, \mathcal{A} \cup \{a : C \sqcap \neg D \geq 1 - x\})$ is satisfiable, where a is a new individual. Therefore, the greatest lower bound problem can be reduced to the minimal satisfiability problem of a fuzzy knowledge base

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Fuzzy OWL

- Fuzzy $\mathcal{SHIF}(D)$, $\mathcal{SHOIN}(D)$, $\mathcal{SROIQ}(D)$, ...
- Additionally, we add
  - modifiers (e.g., very)
  - concrete fuzzy concepts (e.g., Young)
  - both additions have explicit membership functions

Concrete fuzzy concepts

- Examples: Small, Young, High, Tall etc, with explicit membership function
- Use concrete domains to specify them:
  - $D = \langle \Delta_D, \Phi_D \rangle$, where Δ_D is an interpretation domain and Φ_D the set of concrete fuzzy domain predicates d with a predefined arity $n = 1, 2$ and *fixed* interpretation $d^D : \Delta_D^n \to [0, 1]$
- For instance:
  - $\leq_{18}(x)$ over $\mathbb{N}$, evaluates to true if $x \leq 18$, false otherwise, or $cr(0, 18)$
  - Define $Minor = Person \sqcap \exists Age.\leq_{18}$
  - Let $Young : Natural \to [0, 1]$ be a fuzzy datatype predicate denoting the degree of youngness
  - Define $Young(x) = ls(x, 10, 30)$, where ls is the usual left shoulder function
  - Define $YoungPerson = Person \sqcap \exists Age.Young$
  - Then, the KB entails, e.g.:
    $KB \models Minor \sqsubseteq YoungPerson \geq 0.6$,
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  - $D = \langle \Delta_D, \Phi_D \rangle$, where Δ_D is an interpretation domain and Φ_D the set of concrete fuzzy domain predicates d with a predefined arity $n = 1, 2$ and *fixed* interpretation $d^D : \Delta_D^n \rightarrow [0, 1]$
- For instance:
  - $_{\leq 18}(x)$ over $\mathbb{N}$, evaluates to true if $x \leq 18$, false otherwise, or $cr(0, 18)$
  - Define $Minor \equiv Person \sqcap \exists Age._{\leq 18}$
  - Let $Young : Natural \rightarrow [0, 1]$ be a fuzzy datatype predicate denoting the degree of youngness
  - Define $Young(x) = ls(x, 10, 30)$, where ls is the usual left shoulder function
  - Define $YoungPerson \equiv Person \sqcap \exists Age.Young$
  - Then, the KB entails, e.g.:
    $KB \models Minor \sqsubseteq YoungPerson \geq 0.6$,
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Modifiers

- Very, moreOrLess, slightly, etc.
- Apply to fuzzy sets to change their membership function
  - fuzzy modifier m represents a function $f_m : [0,1] \rightarrow [0,1]$, with $\mathbf{M}$ an alphabet for fuzzy modifiers and $m \in \mathbf{M}$
  - then, if C is a concept in, say, fuzzy $\mathcal{SHOIN}$, then so is $m(C)$
  - Modifiers are definable as linear in-equations over $\mathbb{Q}$, $\mathbb{Z}$ (e.g., linear hedges), for instance, linear hedges[4], $lm(x; a, b)$, e.g. $very = lm(x; 0.7, 0.49)$
- Example:
  - $f_{very}(x) = x^2$
  - $f_{slightly}(x) = \sqrt{x}$
  - $SportsCar \equiv Car \sqcap \exists speed.very(High)$, where $very$ is the fuzzy modifier and $High$ a fuzzy datatype over the domain of speed (in km/h) and may be defined as, say, $High(x) = rs(80, 250)$

[4] they modify the shape of a fuzzy set in predictable ways; e.g., by pushing all values less than one towards zero, thereby shrinking the fuzzy part of the set closer to the area that is completely in the set

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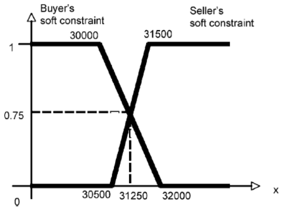
- The car seller and buyer revisited
- More fine grained approach: prices as vague constraints
  - Seller would sell above 31500 euro, but can go down to 30500
  - Buyer prefers to spend less than 30000, but can go up to 32000
    - $AudiTT \sqsubseteq SportsCar \sqcap \exists hasPrice.rs(30500, 31500)$
    - $Request \sqsubseteq SportsCar \sqcap \exists hasPrice.ls(30000, 32000)$
    - with *rs* right-shoulder function and *ls* the left-shoulder function
  - Highest degree to which $C \equiv AudiTT \sqcap Request$ is satisfiable is 0.75 (possibility that the Audi TT and the query matches is 0.75; the $glb(KB, C) = 0.75$)
  - the car may be sold at 31250 euro



Fuzzy OWL and reasoning

- Three principal approaches tested: Tableaux method, MILP based method, MIQP based method
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  - Try to map fuzzy DLs to some fuzzy logic programming: A lot of work exists about mappings among classical DLs and LPs, but needs a theorem prover for fuzzy LPs
  - Build ad-hoc theorem prover for fuzzy DLs, using e.g., MILP
- A theorem prover for fuzzy $\mathcal{SHIF}$ + linear hedges + concrete fuzzy concepts + linear equational constraints + datatypes, under classical, Zadeh, Lukasiewicz and Product t-norm semantics has been implemented (http://gaia.isti.cnr.it/∼straccia)
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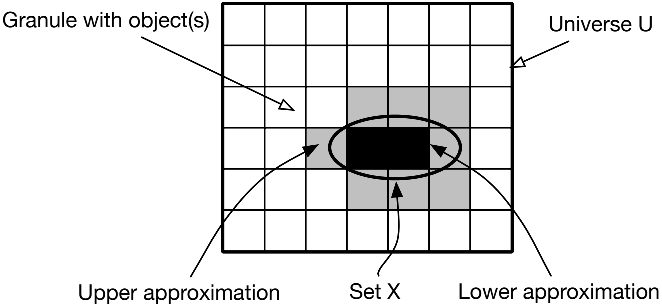
Outline

Rough ontologies: Introduction

- Extension of rough sets to the knowledge representation layer; or: extension of crisp concepts in an ontology to incomplete data in the ABox
- Preliminary results with 'rough ontologies'
- Some results with fuzzy-rough and rough-fuzzy ontology languages
- No (end-user) tools and demonstration case studies in a subject domain yet: requires linking of ontology to sufficient data, i.e. **need for scalable semantic web technologies**

Rough sets

- Brief introduction of the Pawlak rough set model



Rough sets

- $I = (U, A)$ is called an *information system*, where U is a non-empty finite set of objects and A a finite non-empty set of attributes
- For every $a \in A$, function $a : U \mapsto V_a$ where v_a is the set of values that attribute a can have
- For any subset of attributes $P \subseteq A$, one can define the equivalence relation $\text{IND}(P)$ as

$$\text{IND}(P) = \{(x, y) \in U \times U \mid \forall a \in P, a(x) = a(y)\} \tag{8}$$

- $\text{IND}(P)$ generates a partition of U, which is denoted with $U/\text{IND}(P)$, or U/P for short.
- If $(x, y) \in \text{IND}(P)$, then x and y are indistinguishable with respect to the attributes in P, i.e, they are *p-indistinguishable*.

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Rough sets (cont'd)

- From the objects in universe U, we want to represent set X such that $X \subseteq U$ using the attribute set P where $P \subseteq A$
- $[x]_P$ denotes the equivalence classes of the p-indistinguishability relation
- X may not be represented in a crisp way—the set may include and/or exclude objects which are indistinguishable on the basis of the attributes in P—but it can be approximated by using lower and upper approximation, respectively:

$$\underline{P}X = \{x \mid [x]_P \subseteq X\} \quad (9)$$
$$\overline{P}X = \{x \mid [x]_P \cap X \neq \emptyset\} \quad (10)$$

Rough sets (cont'd)

- The *lower approximation* is the set of objects that are *positively* classified as being members of set X, i.e., it is the union of all equivalence classes in $[x]_P$
- The *upper approximation* is the set of objects that are *possibly* in X
- Its complement, $U - \overline{P}X$, is the *negative region* with sets of objects that are definitely not in X (i.e., $\neg X$)
- with every rough set we associate two *crisp* sets, called *lower* and *upper approximation*, denoted as a tuple $X = \langle \underline{X}, \overline{X} \rangle$
- The difference between the lower and upper approximation, $B_P X = \overline{P}X - \underline{P}X$, is the *boundary region* of which its objects neither can be classified as to be member of X nor that they are not in X; if $B_P X = \emptyset$ then X is, in fact, a crisp set with respect to P and when $B_P X \neq \emptyset$ then X is rough w.r.t. P

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Rough ontologies

- Several proposals, mainly DL+ rough extensions
- diverge in the formalisation what to include to represent roughness and thus also as to what rough concepts and rough ontologies actually are
- E is the symmetric, reflexive, transitive equivalence relation
- Let C_R be a (rough) concept in a DL language, then semantics for its lower and upper approximation are:

$$\underline{C} = \{x \mid \forall y : (x, y) \in E \rightarrow y \in C\} \quad (11)$$

$$\overline{C} = \{x \mid \exists y : (x, y) \in E \wedge y \in C\} \quad (12)$$

- Interpretation should map every approximate concept $C_R = \langle \underline{C}, \overline{C} \rangle$ to a pair over $\Delta^{\mathcal{I}}$, i.e., extending $\cdot^{\mathcal{I}}$ as follows:

$$C_R^{\mathcal{I}} = (\langle \underline{C}, \overline{C} \rangle)^{\mathcal{I}} = \langle (\underline{C})^{\mathcal{I}}, (\overline{C})^{\mathcal{I}} \rangle \quad (13)$$

- Interesting property: $C \sqsubseteq D \Rightarrow \langle \underline{C}, \overline{C} \rangle \sqsubseteq \langle \underline{D}, \overline{D} \rangle$

Reasoning services

- Alike the standard DL reasoning services:
  - approximate concept *satisability*, being the definitely satisability and possibly satisability (note that of C_R is possibly unsatisfiable, it is also definitely unsatisfiable)
  - approximate concepts rough *subsumption reasoning*
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Applications

- None, i.e., no use case in a subject domain, except a few toy examples
- Potential: hypothesis testing, classification of patients, etc.
- Extending the theory: fuzzy-rough DL language (Bobillo and Straccia, 2009)
  - Extending also the reasoning algorithms
  - Add $\underline{C}$ and $\overline{C}$ represented as fuzzy DL concepts: $\overline{C}_i \mapsto \exists s_i.C$ and $\underline{C}_i \mapsto \forall s_i.C$, where s_i is a fuzzy similarity relation; add symmetry and reflexivity (in fuzzy $\mathcal{SHOIF}(\mathbf{D})$, i.e. fuzzy OWL Lite)
  - Replace `(upper` s_i `C)` with `(some` s_i `C)` and `(lower` s_i `C)` with `(all` s_i `C)`, then add in the fuzzy RBox the reflexivity, symmetry and transitivity of R (in fuzzy $\mathcal{SROIQ}(\mathbf{D})$, i.e. fuzzy OWL 2 DL)
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Outline

A scenario

Suppose a person would like to "buy a sports car that costs at most about 22 000 euro and that has a power of around 150 HP"

- the buyer has to manually search for car selling web sites, e.g., using Google;
- select the most promising sites;
- browse through them, query them to see the cars that each site sells, and match the cars with the requirements;
- select the offers in each web site that match the requirements; and
- eventually merge all the best offers from each site and select the best ones.

A scenario

- A shopping agent automatizing the whole process once it receives the query q from the buyer:
  - The agent selects some resources S that it considers as relevant to q (*probabilistic*)
  - For the top-k selected sites, the agent reformulates q using the ontology of the specific car selling site (*probabilistic*)
  - q may contain many vague/fuzzy concepts ("around 150 HP"), so a car may *match q to a degree*. So, a resource returns a ranked list of cars, where the ranks depend on the degrees to which the cars match q. (*fuzzy*)
  - The agent integrates the ranked lists (using *probabilities*) and shows the top-n items to the buyer (or divided by *definite* and *possible* matches)
- To do this, there are bits and pieces for the languages and reasoners, but not everything *together*

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  - For the top-k selected sites, the agent reformulates q using the ontology of the specific car selling site (*probabilistic*)
  - q may contain many vague/fuzzy concepts ("around 150 HP"), so a car may *match q to a degree*. So, a resource returns a ranked list of cars, where the ranks depend on the degrees to which the cars match q. (*fuzzy*)
  - The agent integrates the ranked lists (using *probabilities*) and shows the top-n items to the buyer (or divided by *definite* and *possible* matches)
- To do this, there are bits and pieces for the languages and reasoners, but not everything *together*

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Tools

- Probabilistic ontology tools:
  - Pronto: pellet + probabilistic http://pellet.owldl.com/pronto/
  - PR-OWL http://www.pr-owl.org/
  - Probabilisitc Ontology Alignment Tool http://gaia.isti.cnr.it/∼straccia/software/oMap/oMap.html
  - OMEN: A Probabilistic Ontology Mapping Tool
  - BayesOWL, OntoBayes
  - TOSS http://om.umiacs.umd.edu/ptoss.html
- Fuzzy ontology tools:
  - Fuzzy RDF http://gaia.isti.cnr.it/∼straccia/software/fuzzyRDF/fuzzyRDF.html
  - fuzzyDL http://gaia.isti.cnr.it/∼straccia/software/fuzzyDL/fuzzyDL.html
  - DeLorean http://webdiis.unizar.es/∼fbobillo/delorean.php
  - Fuzzy-Kazimir http://www.openclinical.org/prj_kasimir.html
  - FIRE http://www.image.ece.ntua.gr/∼nsimou/

Examples

- Probabilistic ontologies:
  - Star Trek ontology (experimental ontology to demonstrate PR-OWL) `http://www.pr-owl.org/basics/ontostartrek.php`
  - Astronomy to demonstrate TOSS `http://om.umiacs.umd.edu/pparq.html`
- Fuzzy ontologies:
  - Ontology Mediated Multimedia Information Retrieval System `http://gaia.isti.cnr.it/~straccia/software/DL-Media/DL-Media.html`
  - Oncology with FUZZY-KAZIMIR `http://www.oncolor.org/`
  - FIRE with an medical imaging example

Summary

Background

Uncertain knowledge

- Probabilistic logic and ontologies
- Possibilistic logic

Vague Knowledge

- Many-valued logics and ontologies
- Rough sets and ontologies

Tools and applications