First Order Logic – Lab 4

Marijke Keet

KRDB Research Centre, Faculty of Computer Science
Free University of Bozen-Bolzano, Italy
keet@inf.unibz.it

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Midterm comments

- Take care of the **details** and a final answer is not enough. You need to demonstrate—**prove**—why it is that answer.
- Tableaux
Tableaux summary (1/2)

- A sound and complete procedure deciding satisfiability is all we need, and the **tableaux method is a decision procedure which checks the existence of a model**

- It exhaustively looks at all the possibilities, so that it can eventually prove that no model could be found for unsatisfiable formulas.

- $\phi \models \psi$ iff $\phi \land \neg \psi$ is NOT satisfiable—if it is satisfiable, we have found a counterexample

- Decompose the formula in top-down fashion
Tableaux summary (2/2)

- Tableaux calculus works only if the formula has been translated into **Negation Normal Form**, *i.e.*, all the negations have been pushed inside.
- Recollect the list of equivalences, apply those to arrive at NNF, if necessary.
- If a model satisfies a conjunction, then it also satisfies each of the conjuncts.
- If a model satisfies a disjunction, then it also satisfies one of the disjuncts. It is a non-deterministic rule, and it generates two alternative branches.
- Apply the completion rules until either (a) an explicit contradiction due to the presence of two opposite literals in a node (a clash) is generated in each branch, or (b) there is a completed branch where no more rule is applicable.
First order logic: equivalences

- Those from PL + New in FOL: see lecture slides p30. In particular,
  \[ \neg \forall x \phi(x) \equiv \exists x \neg \phi(x) \]
  \[ \neg \exists x \phi(x) \equiv \forall x \neg \phi(x) \]

- The \( \forall \) definition: \( \forall x \phi(x) \) if and only if \( \neg \exists x \neg \phi(x) \)

- Just like with tableaux for PL, we may need them for rewriting a formula suitable for a FOL tableaux
Notes about the midterm recap
Exercises

Equivalences

1. Rewrite $\neg\exists x \forall y (P(x) \rightarrow Q(y))$ into its negation normal form
2. Simplify $\neg\exists x \neg (P(x) \lor Q(x)) \land \forall x (P(x) \rightarrow Q(x))$
3. Are these equivalent/valid? prove it
   - $(\neg\forall y \neg P(y) \lor P(c)) \rightarrow (\forall y \neg P(y) \rightarrow P(c))$
     where $P$ is a unary relation symbol and $c$ a constant symbol
   - $\exists x \phi(x) \land \exists x \psi$ and $\exists x (\phi(x) \land \psi)$
     note that $\phi$, $\exists x \phi$, and $\forall x \phi$ are provably equivalent, and $x$ does not occur as a free variable of $\psi$
Tableaux

- \{\forall x P(x), \exists x (\neg P(x) \lor \neg P(f(c)))\}

More graphs

Consider the following graph, and first-order language $\mathcal{L} = \langle R \rangle$, with $R$ being a binary relation symbol (edge).

1. Formalise the following properties of the graph as $\mathcal{L}$-sentences: (i) $(a, a)$ and $(b, b)$ are edges of the graph; (ii) $(a, b)$ is an edge of the graph; (iii) $(b, a)$ is not an edge of the graph. Let $T$ stand for the resulting set of sentences.

2. Prove that $T \cup \{\forall x \forall y R(x, y)\}$ is unsatisfiable using tableaux calculus.