First Order Logic – Lab 3

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FOL and conceptual data models (optional)

- UML class diagrams in FOL, see e.g. Fig.12 & 13 in §2 Berardi, D., Calvanese, D., De Giacomo, G. Reasoning on UML class diagrams. *Artificial Intelligence*, 2005, 168(1-2): 70-118. http://www.inf.unibz.it/∼calvanese/papers-html/AIJ-2005.html
- Examples of deduction in action (with explanation) in Icom: http://www.inf.unibz.it/∼franconi/icom/tutorial-1.html
- Recollect the ORM diagram in FOL Lab 1

From Natural Language to First order logic

- Each animal is an organism
  All animals are organisms
  If it is an animal then it is an organism
  $\forall x(Animal(x) \rightarrow Organism(x))$
- Each student must be registered for a degree programme
  $\forall x(registered\_for(x, y) \rightarrow Student(x) \wedge DegreeProgramme(y))$
  $\forall x(Student(x) \rightarrow \exists y\ registered\_for(x, y))$
- Aliens exist
  $\exists x\ Alien(x)$
- There are books that are heavy
  $\exists x(Book(x) \wedge heavy(x))$

First order logic: equivalences

- Those from PL + New in FOL: see lecture slides p30[1]
  $\neg\forall x\phi(x) \equiv \exists x\neg\phi(x)$
  $\neg\exists x\phi(x) \equiv \forall x\neg\phi(x)$
- For example:
  - There does not exist a student who is also a professor
    (1) $\neg\exists x(Student(x) \rightarrow Professor(x))$
  - no student is a professor / students are not also professors
    (2) $\forall x\neg(Student(x) \rightarrow Professor(x))$
  - (3) $\neg\exists x\neg\neg(Student(x) \rightarrow Professor(x))$ (∀-Def. in 2)
  - (4) $\neg\exists x(Student(x) \rightarrow Professor(x))$ (double negation 3)
- Of course, you still can play with the formula within the braces
  (a) $\forall x\neg(\neg Student(x) \vee Professor(x))$ (implication 2)
  (b) $\forall x(\neg\neg Student(x) \wedge \neg Professor(x))$ (De Morgan to a)
  (c) $\forall x(Student(x) \wedge \neg Professor(x))$ (Double negation to b)

[1](remember the ∀ definition: $\forall x\phi(x)$ if and only if $\neg\exists x\neg\phi(x)$)

From NL to FOL

1. Pizza is a type of Italian dish
2. Each book must have at least one author
3. Not all humans eat meat
4. For each thymidine phosphorylase, exactly one of the following holds: that thymidine phosphorylase binds some thymidine, that thymidine phosphorylase binds some phosphate.

From NL to FOL

1. Pizza is a type of Italian dish
   $\forall x(Pizza(x) \rightarrow ItalianDish(x))$
2. Each book must have at least one author
   $\forall x(Book(x) \rightarrow \exists y(hasauthor(x, y) \wedge Author(y))$
3. Not all humans eat meat
   $\neg\forall x(Human(x) \rightarrow MeatEater(x))$ (i)
   $\exists x(Human(x) \wedge \neg MeatEater(x))$ (ii)
   $\neg\forall x, y(Human(x) \rightarrow eats(x, y) \wedge Meat(y))$ (iii)
   $\neg\forall x(Human(x) \rightarrow \exists y(eats(x, y) \wedge Meat(y)))$ (iv)
   Is there any difference between (i) & (ii) or between (iii) & (iv)? Prove it.

From NL to FOL

- For each thymidine phosphorylase, exactly one of the following holds: that thymidine phosphorylase binds some thymidine, that thymidine phosphorylase binds some phosphate.
(hint: formalise an exclusive-or constraint)
- Optional: If you like to know more about going from informal to formal, have a look at, e.g.: A.H.M. ter Hofstede, and H.A. (Erik) Proper. (1998). How to Formalize It? Formalization Principles for Information Systems Development Methods. In: *Information and Software Technology*, 40(10), 519-540.
http://citeseer.ist.psu.edu/terhofstede98how.html

Falsifiable sentences

Recollect that ϕ is falsifiable if there is some $(\mathcal{I}, \alpha)$ that does not satisfy ϕ.
Show a counterexample to prove that the following sentences are falsifiable.

1. $\exists x P(x) \rightarrow \forall x P(x)$
2. $\forall x(\phi \vee \psi) \rightarrow \forall x \phi$
3. $\exists x(\phi \vee \psi) \rightarrow \exists x \phi$

Equivalences

1. Rewrite $\neg\exists x \forall y(P(x) \rightarrow Q(y))$ into $\forall x \exists y(P(x) \land \neg Q(y))$
2. Simplify $\neg\exists x \neg(P(x) \lor Q(x)) \land \forall x(P(x) \rightarrow Q(x))$
3. Are these equivalent/valid? prove it
   - $(\neg\forall y \neg P(y) \lor P(c)) \rightarrow (\forall y \neg P(y) \rightarrow P(c))$
     where P is a unary relation symbol and c a constant symbol
   - $\exists x \phi(x) \land \exists x \psi$ and $\exists x(\phi(x) \land \psi)$
     note that φ, $\exists x \varphi$, and $\forall x \varphi$ are provably equivalent, and x does not occur as a free variable of ψ
4. ...