Recap

Exercises

FOL and conceptual data models (optional)


- Examples of deduction in action (with explanation) in Icom: http://www.inf.unibz.it/~franconi/icom/tutorial-1.html

- Recollect the ORM diagram in FOL Lab 1
Each animal is an organism
All animals are organisms
If it is an animal then it is an organism
\( \forall x (Animal(x) \rightarrow Organism(x)) \)

Each student must be registered for a degree programme
\( \forall x (\text{registered\_for}(x, y) \rightarrow Student(x) \land DegreeProgramme(y)) \)
\( \forall x (Student(x) \rightarrow \exists y \text{ registered\_for}(x, y)) \)

Aliens exist
\( \exists x \ Alien(x) \)

There are books that are heavy
\( \exists x (Book(x) \land heavy(x)) \)
First order logic: equivalences

- Those from PL + New in FOL: see lecture slides p30\(^1\)
  \(-\forall x \phi(x) \equiv \exists x \neg \phi(x)\)
  \(-\exists x \phi(x) \equiv \forall x \neg \phi(x)\)

- For example:
  - There does not exist a student who is also a professor
    \((1) \quad \neg \exists x (\text{Student}(x) \rightarrow \text{Professor}(x))\)
  - no student is a professor / students are not also professors
    \((2) \quad \forall x \neg (\text{Student}(x) \rightarrow \text{Professor}(x))\)
  - (3) \quad \neg \exists x \neg (\text{Student}(x) \rightarrow \text{Professor}(x)) \quad \text{(\forall\text{-Def. in 2})}
  - (4) \quad \neg \exists x (\text{Student}(x) \rightarrow \text{Professor}(x)) \quad \text{(double negation 3)}

- Of course, you still can play with the formula within the braces
  - (a) \quad \forall x \neg (\neg \text{Student}(x) \lor \text{Professor}(x)) \quad \text{(implication 2)}
  - (b) \quad \forall x (\neg \neg \text{Student}(x) \land \neg \text{Professor}(x)) \quad \text{(De Morgan to a)}
  - (c) \quad \forall x (\text{Student}(x) \land \neg \text{Professor}(x)) \quad \text{(Double negation to b)}

\(^1\text{(remember the \forall definition: } \forall x \phi(x) \text{ if and only if } \neg \exists x \neg \phi(x)\)
From NL to FOL

1. Pizza is a type of Italian dish
2. Each book must have at least one author
3. Not all humans eat meat
4. For each thymidine phosphorylase, exactly one of the following holds: that thymidine phosphorylase binds some thymidine, that thymidine phosphorylase binds some phosphate.
1. Pizza is a type of Italian dish
   \( \forall x (\text{Pizza}(x) \rightarrow \text{ItalianDish}(x)) \)

2. Each book must have at least one author
   \( \forall x (\text{Book}(x) \rightarrow \exists y (\text{hasauthor}(x, y) \land \text{Author}(y)) ) \)

3. Not all humans eat meat
   \( \neg \forall x (\text{Human}(x) \rightarrow \text{MeatEater}(x)) \) \hspace{1cm} (i)
   \( \exists x (\text{Human}(x) \land \neg \text{MeatEater}(x)) \) \hspace{1cm} (ii)
   \( \neg \forall x, y (\text{Human}(x) \rightarrow \text{eats}(x, y) \land \text{Meat}(y)) \) \hspace{1cm} (iii)
   \( \neg \forall x (\text{Human}(x) \rightarrow \exists y (\text{eats}(x, y) \land \text{Meat}(y))) \) \hspace{1cm} (iv)

Is there any difference between (i) & (ii) or between (iii) & (iv)? Prove it.
For each thymidine phosphorylase, exactly one of the following holds: that thymidine phosphorylase binds some thymidine, that thymidine phosphorylase binds some phosphate. (hint: formalise an exclusive-or constraint)

Recall that \( \phi \) is falsifiable if there is some \((\mathcal{I}, \alpha)\) that does not satisfy \( \phi \).

Show a counterexample to prove that the following sentences are falsifiable.

1. \( \exists x P(x) \rightarrow \forall x P(x) \)
2. \( \forall x (\phi \lor \psi) \rightarrow \forall x \phi \)
3. \( \exists x (\phi \lor \psi) \rightarrow \exists x \phi \)
Rewrite \( \neg \exists x \forall y (P(x) \to Q(y)) \) into \( \forall x \exists y (P(x) \land \neg Q(y)) \)

Simplify \( \neg \exists x \neg (P(x) \lor Q(x)) \land \forall x (P(x) \to Q(x)) \)

Are these equivalent/valid? prove it

1. \((\neg \forall y \neg P(y) \lor P(c)) \to (\forall y \neg P(y) \to P(c))\)
2. \(\exists x \phi(x) \land \exists x \psi\) and \(\exists x (\phi(x) \land \psi)\)

where \(P\) is a unary relation symbol and \(c\) a constant symbol

note that \(\phi, \exists x \phi,\) and \(\forall x \phi\) are provably equivalent, and \(x\) does not occur as a free variable of \(\psi\)

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