First Order Logic – Lab 1

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From Lab 3:

- \((A \land B) \lor C \models (A \rightarrow \neg B) \rightarrow C\) (equivalences exercise)
- \(((A \rightarrow B) \land (C \rightarrow \neg D)) \rightarrow (C \rightarrow \neg B)\) (the shaving story)
Student **is an entity type.**
DegreeProgramme **is an entity type.**
Student attends DegreeProgramme.

**Each** Student attends *exactly one* DegreeProgramme.
**It is possible that more than one** Student attends the same DegreeProgramme.
**OR, in the negative:**
**For each** Student, *it is impossible that that* Student attends *more than one* DegreeProgramme.
**It is impossible that any** Student attends *no* DegreeProgramme.

<table>
<thead>
<tr>
<th>Student</th>
<th>DegreeProgramme</th>
</tr>
</thead>
<tbody>
<tr>
<td>John</td>
<td>Computer Science</td>
</tr>
<tr>
<td>Mary</td>
<td>Design</td>
</tr>
<tr>
<td>Fabio</td>
<td>Design</td>
</tr>
<tr>
<td>Claudio</td>
<td>Computer Science</td>
</tr>
<tr>
<td>Markus</td>
<td>Biology</td>
</tr>
<tr>
<td>Inge</td>
<td>Computer Science</td>
</tr>
</tbody>
</table>
Examples of first-order structures

- Graphs are mathematical structures.
- A graph is a set of points, called **vertices**, and lines, called **edges** between them. For instance:

  ![Graphs A, B, and C](image)

  - Figures A and B are different depictions, but have the same descriptions w.r.t. the vertices and edges. Check this.
  - Graph C has a property that A and B do not have. Represent this in a first-order sentence.
  - Find a suitable first-order language for A (B), and formulate at least two properties of the graph using quantifiers.
Checking

Consider a first order language where \( R \) is a binary relation symbol and \( P \) a unary relation symbol (UML class, ER entity type, ORM object type) and an interpretation \( \mathcal{I} \) with domain \( \{0, 1\} \), where:

\[
P^\mathcal{I} = \{0, 1\} \quad (1)
\]
\[
R^\mathcal{I} = \{(0, 0), (0, 1)\} \quad (2)
\]

Check whether \( \mathcal{I} \) is a model of the following formulas:

\[
\forall x \exists y R(x, y) \quad (3)
\]
\[
\exists x \forall y R(x, y) \quad (4)
\]
\[
\forall x P(x) \quad (5)
\]
\[
\exists x P(x) \quad (6)
\]
\[
\forall x \forall y (R(x, y) \lor P(x)) \quad (7)
\]
\[
\forall x \forall y (R(x, y) \land (P(x) \lor P(y))) \quad (8)
\]