Propositional Logic – Lab 2

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Some definitions

- A formula is valid if it holds under *every* assignment. $\models F$ to denote this. A valid formula is called a tautology.
- A formula is satisfiable if it holds under *some* assignment.
- A formula is unsatisfiable if it holds under *no* assignment. An unsatisafiable formula is called a contradiction.

Ordering

- The order of things: $\neg$ has priority over $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$.
- For the rest, it is better to use braces to group formulas.
- Compare, e.g., $A \wedge (B \rightarrow C)$ with $(A \wedge B) \rightarrow C$ and note the **differences in satisfiable truth assignments** in their respective truth table.

Implication and their English counterparts

A	B	$A \rightarrow B$ $\neg A \lor B$	can read it as	
			If A then B	B follows from A
0	0	1	A implies B	A is sufficient for B
0	1	1	A only if B	B is necessary for A
1	0	0	B if A	B is a necessary condition for A
1	1	1	Whenever A, B	B whenever A
			Not A unless B	A is a sufficient condition for B

Tibbles

- Is the following argument valid? Represent the argument formally and use truth tables to prove it.
  - If Tibbles roves the Lungo Talvera, he lives in Bolzano.
  - Tibbles lives in Bolzano.
  - Therefore Tibbles roves the Lungo Talvera.
  - $((A \rightarrow B) \land B) \rightarrow A$

Truth tables & look ahead

- Find the truth tables for each of the following formulas and state whether each is a tautology, a contradiction, or neither.
  - $(\neg A \rightarrow B) \vee ((A \wedge \neg C) \leftrightarrow B)$
- Is the following formula satisfiable, valid, a contradiction, or a tautology?
  - $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$
  - Hint: check the list of equivalences

Entailment

- Truth value assignment (interpretation) of all atoms in Σ is a function $\mathcal{I}$ where $\mathcal{I} : \Sigma \rightarrow \{T, F\}$
- An interpretation $\mathcal{I}$ is a model of ϕ, written as $\mathcal{I} \models \phi$.
- We can do the same for *sets* of formulas Θ; i.e., $\mathcal{I} \models \Theta$ iff $\mathcal{I} \models \phi$ for all $\phi \in \Theta$
- We want formula ϕ to be implied by Θ, if ϕ is true in all models of Θ, written as $\Theta \models \phi$, so we get $\Theta \models \phi$ iff $\mathcal{I} \models \phi$ for all models $\mathcal{I}$ of Θ
- Properties of entailment.
    - Deduction theorem: $\Theta \cup \{\phi\} \models \psi$ iff $\Theta \models \phi \rightarrow \psi$
    - Contraposition theorem: $\Theta \cup \{\phi\} \models \neg\psi$ iff $\Theta \cup \{\psi\} \models \neg\phi$
    - Contradiction theorem: $\Theta \cup \{\phi\}$ is unsatisfiable iff $\Theta \models \neg\phi$

Entailment

- E.g. Θ is knowledge base, KB, which contains $(A \vee C) \wedge (B \vee \neg C)$, and we want to know if formula ϕ holds, where $\phi = A \vee B$, i.e. $KB \models \phi$?
  - Hint: check all possible models: ϕ must be true wherever KB is true.... truth table.
- $\models \neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$
- $\models \neg(A \rightarrow B) \leftrightarrow (\neg A \vee B)$
- $\models (A \rightarrow (B \wedge C)) \rightarrow (\neg(B \wedge C) \rightarrow \neg A)$
  - Hint: you can do it with truth tables, but have a look at the equivalences, too.

Equivalence

- Which one(s) of the following is (are) equivalent?
  - $(((A \rightarrow B) \rightarrow B) \rightarrow B)$ and $(A \rightarrow B)$
  - $(A \wedge B) \vee C$ and $(A \rightarrow \neg B) \rightarrow C$
    - hint: for $(A \wedge B) \vee C$, try distributivity, or for $(A \rightarrow \neg B) \rightarrow C$, try implication
    - which gives: $(A \vee C) \wedge (B \vee C)$, resp. $\neg(A \rightarrow \neg B) \vee C$
    - more implication gives $\neg(\neg A \vee \neg B) \vee C$
    - hint: De Morgan.
    - $(\neg\neg A \wedge \neg\neg B) \vee C$
    - Double negation, then $(A \wedge B) \vee C$