

# Representing and reasoning over a taxonomy of part-whole relations

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- 1 Background
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- 2 Taxonomy of types of part-whole relations
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- 4 Orthogonal and interfering subtopics
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# Some questions and problems (not exhaustive...)

- Is a tunnel part of the mountain?
- What is the difference, if any, between how Cell nucleus and Cell are related and how Receptor and Cell wall are related?
- And w.r.t. Brain part of Human and/versus Hand part of Boxer? (assuming boxers must have their own hands)
- A classical example: hand is part of musician, musician part of orchestra, but clearly, the musician's hands are not part of the orchestra. Is part-of then not transitive, or is there a problem with the example?

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# Analysis of the issues from diverse angles

- Mereological theories [Varzi04], usage & extensions (e.g. mereotopology, relation with granularity, set theory)
- Early attempts with direct parthood [Sattler95], SEP triples [SH00], and other outstanding issues [AFGP96], some still remaining [KA08].
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# Ground Mereology

Reflexivity (everything is part of itself)

$$\forall x(part\_of(x, x)) \quad (1)$$

Antisymmetry (two distinct things cannot be part of each other, or: if they are, then they are the same thing)

$$\forall x, y((part\_of(x, y) \wedge part\_of(y, x)) \rightarrow x = y) \quad (2)$$

Transitivity (if x is part of y and y is part of z, then x is part of z)

$$\forall x, y, z((part\_of(x, y) \wedge part\_of(y, z)) \rightarrow part\_of(x, z)) \quad (3)$$

Proper parthood

$$\forall x, y(proper\_part\_of(x, y) \equiv part\_of(x, y) \wedge \neg part\_of(y, x)) \quad (4)$$

# Ground Mereology

## Proper parthood

$$\forall x, y (proper\_part\_of(x, y) \equiv part\_of(x, y) \wedge \neg part\_of(y, x)) \quad (5)$$

## Asymmetry (if $x$ is part of $y$ then $y$ is not part of $x$ )

$$\forall x, y (part\_of(x, y) \rightarrow \neg part\_of(y, x)) \quad (6)$$

## Irreflexivity ( $x$ is not part of itself)

$$\forall x \neg (part\_of(x, x)) \quad (7)$$

# Defining other relations with *part\_of*

Overlap (x and y share a piece z)

$$\forall x, y (\text{overlap}(x, y) \equiv \exists z (\text{part\_of}(z, x) \wedge \text{part\_of}(z, y))) \quad (8)$$

Underlap (x and y are both part of some z)

$$\forall x, y (\text{underlap}(x, y) \equiv \exists z (\text{part\_of}(x, z) \wedge \text{part\_of}(y, z))) \quad (9)$$

Over- & undercross (over/underlap but not part of)

$$\forall x, y (\text{overcross}(x, y) \equiv \text{overlap}(x, y) \wedge \neg \text{part\_of}(x, y)) \quad (10)$$

$$\forall x, y (\text{undercross}(x, y) \equiv \text{underlap}(x, y) \wedge \neg \text{part\_of}(y, x)) \quad (11)$$

Proper overlap & Proper underlap

$$\forall x, y (\text{p\_overlap}(x, y) \equiv \text{overcross}(x, y) \wedge \text{overcross}(y, x)) \quad (12)$$

$$\forall x, y (\text{p\_underlap}(x, y) \equiv \text{undercross}(x, y) \wedge \text{undercross}(y, x)) \quad (13)$$

- With  $x$  as part, what to do with the remainder that makes up  $y$ ?
  - Weak supplementation: every proper part must be supplemented by another, disjoint, part. **MM**
  - Strong supplementation: if an object fails to include another among its parts, then there must be a remainder. **EM**
- Problem with EM: non-atomic objects with the same proper parts are identical, because of this (extensionality principle), but sameness of parts may not be sufficient for identity E.g.: two objects can be distinct purely based on arrangement of its parts, differences statue and its marble (multiplicative approach)

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# General Extensional Mereology

- Strong supplementation [EM]

$$\neg \text{part\_of}(y, x) \rightarrow \exists z(\text{part\_of}(z, y) \wedge \neg \text{overlap}(z, x)) \quad (14)$$

- And add unrestricted fusion [GEM]. Let  $\phi$  be a property or condition, then for every satisfied  $\phi$  there is an entity consisting of all entities that satisfy  $\phi$ .<sup>1</sup> Then:

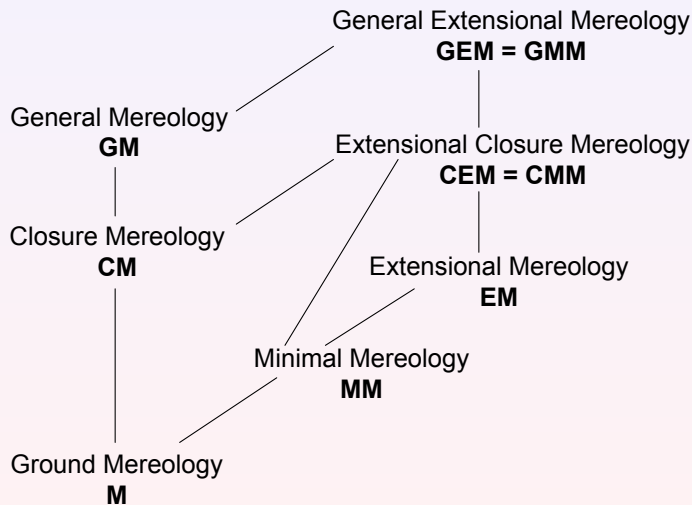
$$\exists x\phi \rightarrow \exists z\forall y(\text{overlap}(y, z) \leftrightarrow \exists x(\phi \wedge \text{overlap}(y, x))) \quad (15)$$

- Note that in EM and upward we have identity, from which one can prove acyclicity for ppo
- There are more mereological theories, and the above is not uncontested (more about that later)

---

<sup>1</sup>Need to refer to classes, but desire to stay within FOL. Solution: axiom schema with only predicates or open formulas

# Relations between common mereological theories





*Can any of this be represented in a decidable fragment of first order logic for use in ontologies and (scalable) software implementations?*

# Things are improving...

- Early days (90s) and simplest options: DL-role  $R$  as partof, or has-part added as primitive role as  $\succeq$ , model it as the transitive closure of a parthood relation (16) and define e.g. Car as having wheels that in turn have tires [AFGP96] (17):

$$\succeq \doteq (\text{primitive-part}) * \quad (16)$$

$$\text{Car} \doteq \exists \succeq . (\text{Wheel} \sqcap \exists \succeq . \text{Tire}) \quad (17)$$

Then  $\text{Car} \sqsubseteq \exists \succeq . \text{Tire}$

- SEP triples with  $\mathcal{ALC}$  [SH00]
  - What  $\mathcal{SHIQ}$  fixes cf.  $\mathcal{ALC}$ : Transitive roles, Inverse roles (to have both part-of and has-part), Role hierarchies (e.g. for subtypes of part-of), qualified Number restrictions (e.g. to represent that a bicycle has-part 2 wheels)
  - Build-your-own DL-language [BD05]

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# What we can(not) implement now with DL-based ontology languages

**Table:** Properties of parthood and proper parthood compared to their support in  $\mathcal{DLR}_\mu$ ,  $\mathcal{SHOIN}$  and  $\mathcal{SROIQ}$ . \*: properties of the parthood relation (in M); †: properties of the proper parthood relation (in M).

Language $\Rightarrow$ Feature $\Downarrow$	$\mathcal{DLR}_\mu$	$\mathcal{SHOIN}$ ( $\sim$ OWL-DL)	$\mathcal{SROIQ}$ ( $\sim$ OWL 2)	DL-Lite <sub>A</sub>
Reflexivity *	+	–	+	–
Antisymmetry *	–	–	–	–
Transitivity * †	+	+	+	–
Asymmetry †	+	+	+	+
Irreflexivity †	+	–	+	–
Acyclicity	+	–	–	–

# Definitions in OBO Relations Ontology [Setal05]

- Instance-level relations
  - $c$  **part\_of**  $c_1$  at  $t$  - a primitive relation between two continuant instances and a time at which the one is part of the other
  - $p$  **part\_of**  $p_1$ ,  $r$  **part\_of**  $r_1$  - a primitive relation of parthood, holding independently of time, either between process instances (one a subprocess of the other), or between spatial regions (one a subregion of the other)
  - $c$  **contained\_in**  $c_1$  at  $t \triangleq c$  **located\_in**  $c_1$  at  $t$  and not  $c$  **overlap**  $c_1$  at  $t$
  - $c$  **located\_in**  $r$  at  $t$  - a primitive relation between a continuant instance, a spatial region which it occupies, and a time

# Definitions in OBO Relations Ontology [Setal05]

- Class-level relations
  - $C \text{ part\_of } C_1 \triangleq$  for all  $c, t$ , if  $Cct$  then there is some  $c_1$  such that  $C_1c_1t$  and  $c$  **part\_of**  $c_1$  **at**  $t$ .
  - $P \text{ part\_of } P_1 \triangleq$  for all  $p$ , if  $Pp$  then there is some  $p_1$  such that:  $P_1p_1$  and  $p$  **part\_of**  $p_1$ .
  - $C \text{ contained\_in } C_1 \triangleq$  for all  $c, t$ , if  $Cct$  then there is some  $c_1$  such that:  $C_1c_1t$  and  $c$  **contained\_in**  $c_1$  **at**  $t$
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Label the relations differently

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# Linguistic use of part-whole relations (meronymy)

- Part of?
  - ★ Centimeter part of Decimeter
  - ★ Decimeter part of Meter
  - *therefore* Centimeter part of Meter
  - ★ Meter part of SI
  - but *not* Centimeter part of SI
- Transitivity?
  - ★ Person part of Organisation
  - ★ Organisation located in Bolzano
  - *therefore* Person located in Bolzano?
  - but *not* Person part of Bolzano

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  - but *not* Person **member of** Bolzano

# Linguistic use of part-whole relations

- Which part of?
  - ★ CellMembrane structural part of RedBloodCell
  - ★ RedBloodCell part of Blood
    - but *not* CellMembrane structural part of Blood
  - ★ Receptor structural part of CellMembrane
    - *therefore* Receptor structural part of RedBloodCell

# Linguistic use of part-whole relations

- Which part of?
  - ★ CellMembrane structural part of RedBloodCell
  - ★ RedBloodCell **contained in?** Blood
    - but *not* CellMembrane structural part of Blood
  - ★ Receptor structural part of CellMembrane
    - *therefore* Receptor structural part of RedBloodCell

# Addressing the issues

- Efforts to disambiguate this confusion; e.g. an informal taxonomy [WCH87] list of 6 types motivated by CMing [Odell98] [GP95] ontology-inspired CMing [G05]
- Location, containment, membership of a collective, quantities of a mass
- Relatively well-settled debate on transitivity, or not [Varzi06] and related papers in AO

# Overview

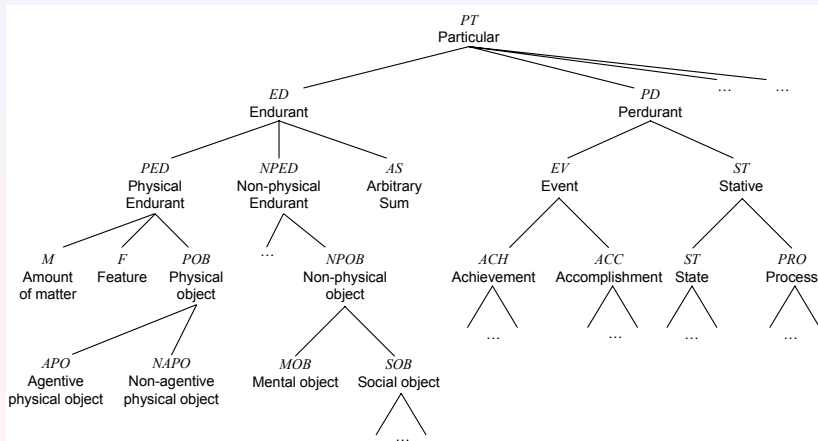
- Mereological *part\_of* (and subtypes) versus 'other' part-whole relations
- Categories of object types of the part-whole relation changes
- Structure these relations by (non/in)transitivity and kinds of relata
- Simplest mereological theory, M.
- Commit to a foundational ontology: DOLCE [MBGG003] (though one also could choose, a.o., BFO, OCHRE, GFO, ...)

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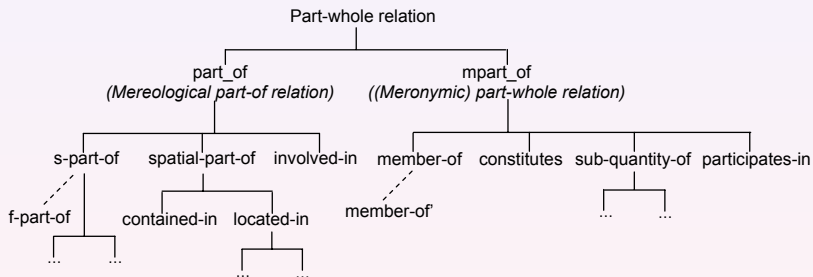
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# DOLCE categories



# Part-whole relations



# Part-whole relations

“member-bunch”, collective nouns (e.g. Herd, Orchestra) with their members (Sheep, Musician)

$$\forall x, y (member\_of_n(x, y) \triangleq mpart\_of(x, y) \wedge (POB(x) \vee SOB(x)) \wedge SOB(y))$$

“material-object”, that what something is made of (e.g., Vase and Clay)

$$\forall x, y (constitutes_{it}(x, y) \equiv constituted\_of_{it}(y, x) \triangleq mpart\_of(x, y) \wedge POB(y) \wedge M(x))$$

## Part-whole relations

“quantity-mass”, “portion-object”, relating a smaller (or sub) part of an amount of matter to the whole. Two issues (glass of wine & bottle of wine vs. Salt as subquantity of SeaWater)

$$\forall x, y (sub\_quantity\_of_n(x, y) \triangleq mpart\_of(x, y) \wedge M(x) \wedge M(y))$$

“noun-feature/activity”, entity participates in a process, like Enzyme that participates in CatalyticReaction

$$\forall x, y (participates\_in_{it}(x, y) \triangleq mpart\_of(x, y) \wedge ED(x) \wedge PD(y))$$

# Part-whole relations

processes and sub-processes (e.g. Chewing is involved in the grander process of Eating)

$$\forall x, y (involved\_in(x, y) \triangleq part\_of(x, y) \wedge PD(x) \wedge PD(y))$$

Object and its 2D or 3D region, such as contained\_in(John's address book, John's bag) and located\_in(Pretoria, South Africa)

$$\begin{aligned} \forall x, y (contained\_in(x, y) \triangleq & part\_of(x, y) \wedge R(x) \wedge R(y) \wedge \\ & \exists z, w (has\_3D(z, x) \wedge has\_3D(w, y) \wedge ED(z) \wedge ED(w))) \end{aligned}$$

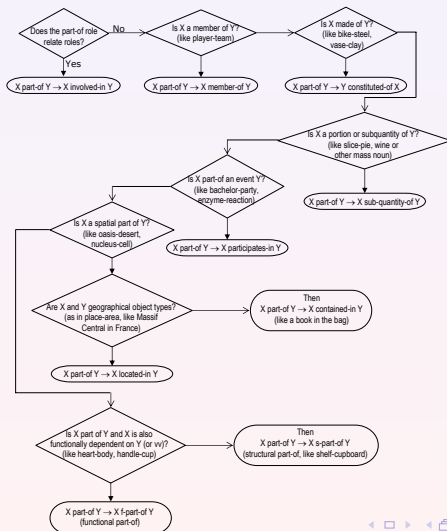
$$\begin{aligned} \forall x, y (located\_in(x, y) \triangleq & part\_of(x, y) \wedge R(x) \wedge R(y) \wedge \\ & \exists z, w (has\_2D(z, x) \wedge has\_2D(w, y) \wedge ED(z) \wedge ED(w))) \end{aligned}$$

$$\forall x, y (s\_part\_of(x, y) \triangleq part\_of(x, y) \wedge ED(x) \wedge ED(y))$$

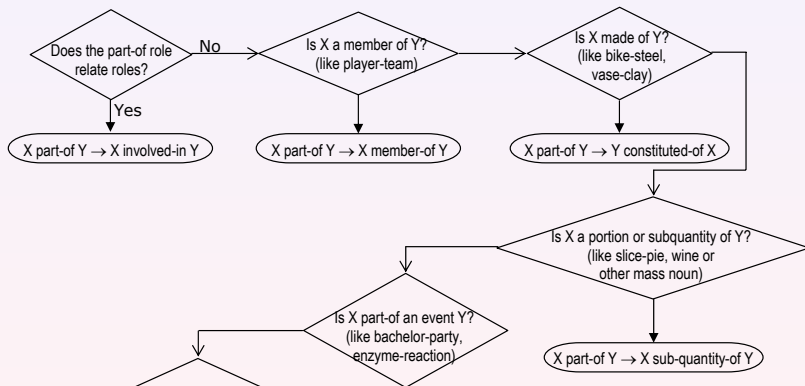
# Using the taxonomy of part-whole relations

- Representing it correctly in ontologies and conceptual data models
- Reasoning with a taxonomy of relations

# Decision diagram [K06a]

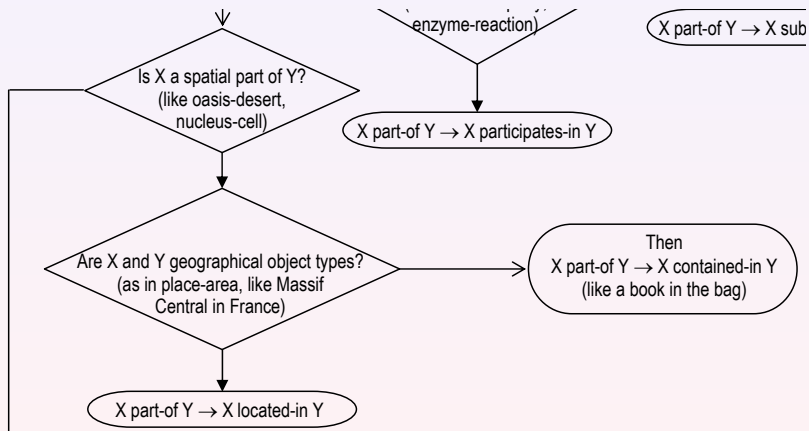


## Decision diagram

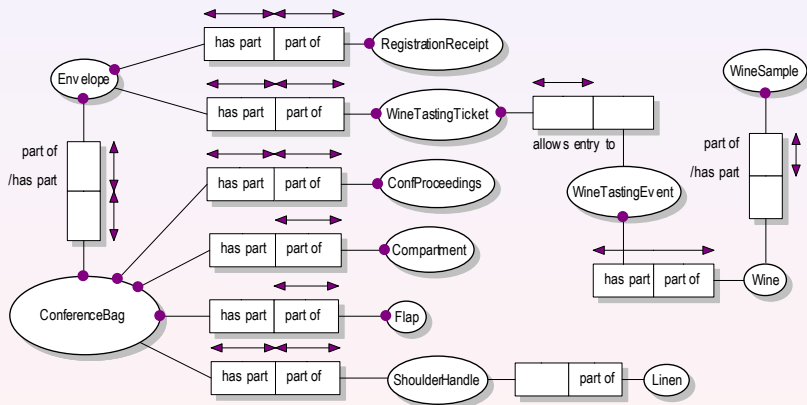




# Decision diagram



## Example - before



## Example - after

- Envelope is not involved-in, not a member-of, does not constitute, is not a sub-quantity of, does not participate-in, is not a geographical object, but instead is contained-in the ConferenceBag.
- Transitivity holds for the mereological relations: derived facts are automatically correct, like RegistrationReceipt contained-in ConferenceBag.
- Intransitivity of Linen and ConferenceBag, because a conference bag is not wholly constituted of linen (the model does not say what the Flap is made of).
- Completeness, i.e. that *all* parts make up the whole, is implied thanks to the closed-world assumption. ConferenceBag directly contains the ConfProceedings and Envelope *only*, and does not contain, say, the Flap.

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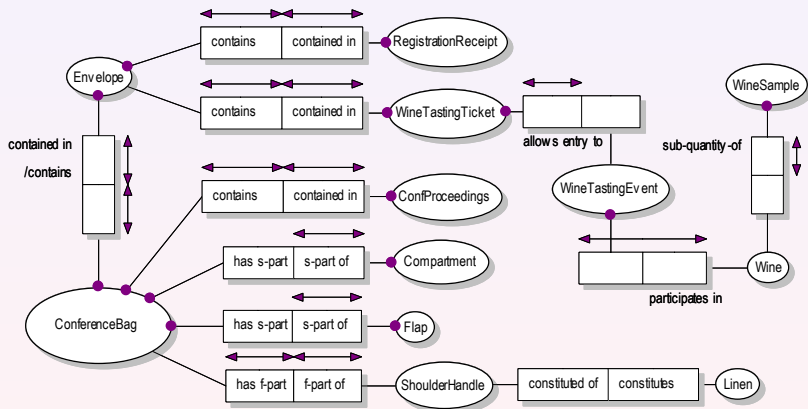
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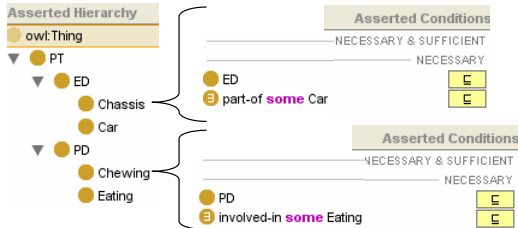
# Requirements

- Represent at least Ground Mereology,
- Express ontological categories and their taxonomic relations,
- Having the option to represent transitive and intransitive relations, and
- Specify the domain and range restrictions (/relata/entity types) for the classes participating in a relation.



# Current behaviour of reasoners

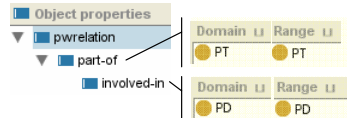
## A1. Class hierarchy with asserted conditions



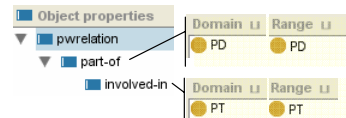
## A2. Other class hierarchy with the same asserted conditions



## B. Correct role box (object properties)



## C. Wrong role box (object properties)

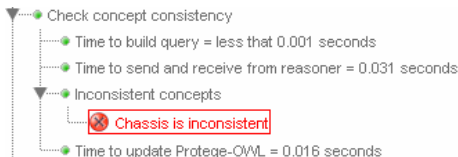


# Current behaviour of reasoners

1. A1+B+racer: *ontology OK*

2. A2+B+racer: *ontology OK*

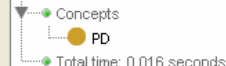
3. A1+C+racer: class hierarchy is inconsistent



4. A2+C+racer: Chassis reclassified as PD

Computing superclasses: Querying reasoner...

Reasoner log



# The *RBox Compatibility* service – definitions

## Definition (Domain and Range Concepts)

Let  $R$  be a role and  $R \sqsubseteq C_1 \times C_2$  its associated Domain & Range axiom. Then, with the symbol  $D_R$  we indicate the *User-defined Domain* of  $R$ —i.e.,  $D_R = C_1$ —while with the symbol  $R_R$  we indicate the *User-defined Range* of  $R$ —i.e.,  $R_R = C_2$ .

## Definition (RBox Compatibility)

For each pair of roles,  $R, S$ , such that  $\langle \mathcal{T}, \mathcal{R} \rangle \models R \sqsubseteq S$ , check:

- Test 1.  $\langle \mathcal{T}, \mathcal{R} \rangle \models D_R \sqsubseteq D_S$  and  $\langle \mathcal{T}, \mathcal{R} \rangle \models R_R \sqsubseteq R_S$ ;
- Test 2.  $\langle \mathcal{T}, \mathcal{R} \rangle \not\models D_S \sqsubseteq D_R$ ;
- Test 3.  $\langle \mathcal{T}, \mathcal{R} \rangle \not\models R_S \sqsubseteq R_R$ .

An RBox is said to be compatible iff *Test 1* and (2 or 3) hold for all pairs of role-subrole in the RBox.

## The *RBox Compatibility* service – behaviour

- If Test 1 does not hold: warning that domain & range restrictions of either  $R$  or  $S$  are in conflict with the role hierarchy proposing either
  - (i) To change the role hierarchy or
  - (ii) To change domain & range restrictions or
  - (iii) If the test on the domains fails, then propose a new axiom  $R \sqsubseteq D'_R \times R_R$ , where  $D'_R \equiv D_R \sqcap D_S^2$ , which subsequently has to go through the RBox compatibility service (and similarly when Test 1 fails on range restrictions).

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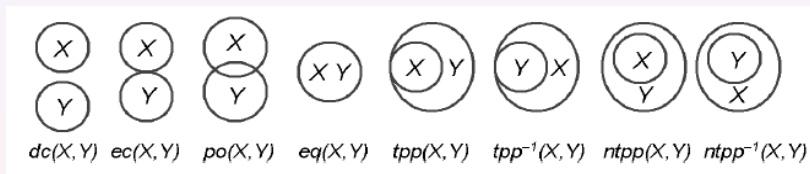
<sup>2</sup>The axiom  $C_1 \equiv C_2$  is a shortcut for the axioms:  $C_1 \sqsubseteq C_2$  and  $C_2 \sqsubseteq C_1$ .

## The *RBox Compatibility* service – behaviour

- If Test 2 and Test 3 fail: warn that  $R$  cannot be a proper subrole of  $S$  but that the two roles can be equivalent. Then, either:
  - (a) Accept the possible equivalence between the two roles or
  - (b) Change domain & range restrictions.
- Ignoring all warnings is allowed, too

## Extensions in various directions

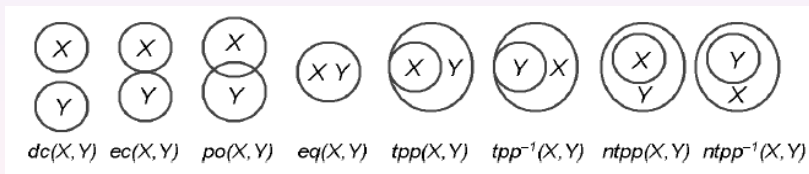
- Mereotopology, with location, GIS, Region Connection Calculus (<http://www.comp.leeds.ac.uk/qsr/rcc.html>)



- Mereogeometry [BM07]
  - Mereology and/vs granularity (a.o. [BS03], [K08], [RRB06])
  - Temporalising the part-whole relations (a.o. [BD07] [AGK08])

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# Some more issues in parthood relations

- What to do with other (meta-)properties, such as:
  - **Inseparability** ( $x$  is inseparable of  $y$  iff  $x$  is existentially dependent on  $y$  and  $x$  is necessarily a part of  $y$ , see [G05] p169)
  - Degrees of **shareability** ('total', part of more than whole of the same type or of different types, etc. [MK99])
  - **Essential part** & essential whole (like member-partnership [Odell98], brain-human)
  - **Immutable part** & immutable whole (the boxer with his hands, an ecofarm with a piece of farmland)
- **De dicto/de re distinction** (possible worlds) [G07], where
  - de re with  $\forall... \rightarrow \Box...$ , e.g. "every boxer necessarily has a hand"
  - de dicto with  $\Box(\forall... \rightarrow ...)$  e.g. "necessarily, every boxer has a hand"
- **Total/complete** on the parts
- **Direct part** to distinguish from part-by-transitivity



# Some more issues in parthood relations

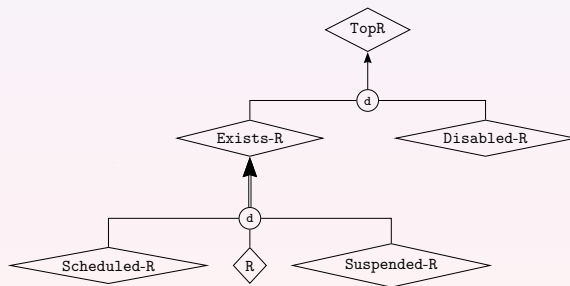
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# (Non-)Sharable parts and wholes

- Essential and mandatory parts; informally, distinguish between:
  - “must be part of *some* whole” and
  - “must be part of *the same* whole” during its lifetime.
- Sequentially or concurrently being part of  $> 1$  whole of same or different type; distinguish between, a.o.:
  - $p$  is part of  $w_1 \in W$  at time  $t_1$  and of  $w_2 \in W$  at time  $t_2$ ;
  - $p$  is part of  $w_1 \in W$  and  $w_2 \in W$  at time  $t_1$ ;
  - $p$  is part of  $w_1 \in W$  at time  $t_1$  and of  $w_a \in W'$  at time  $t_2$ ;
  - $p$  is part of  $w_1 \in W$  and  $w_a \in W'$  at time  $t_1$ ;

# Solution sketch

- Temporalizing part-whole relations, and parts and wholes
- $\mathcal{DLR}_{US}$  and  $\mathcal{ER}_{VT}$  [APS07], extended with *status relations* (details in [AGK08])



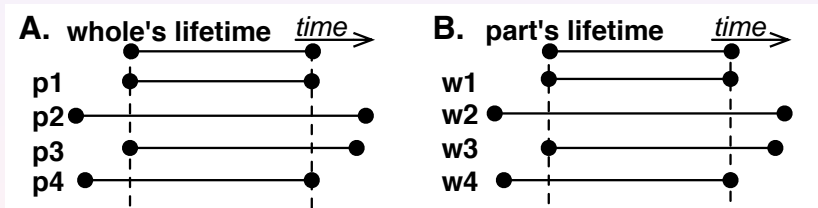
# Examples

- **Scheduled**: a relation is scheduled if its instantiation is known but its membership will only become effective some time later. For instance, a pillar for finishing the interior of the Sagrada Familia in Barcelona is scheduled to become part of that church.
- **Active**: the status of a relation is active if the particular relation fully instantiates the type-level relation. For instance, the Mont Blanc mountain is part of the Alps mountain range, and the country Republic of Ireland is part of the European Union.

# Examples

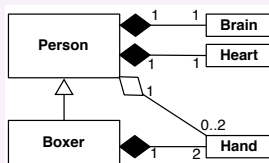
- **Suspended**: to capture a temporarily inactive relation. For example, an instance of a CarEngine is removed from the instance of a Car it is part of, for purpose of maintenance at the car mechanic.
- **Disabled**: to model expired relations that never again can be used. For instance, to represent the donor of an organ who has donated that organ and one wants to keep track of who donated what to whom: say, the heart  $p_1$  of donor  $w_1$  used to be a structural part of  $w_1$  but it will never be again a part of it. The heart,  $p_1$ , then may have become participant in a new part-of relation with a new whole,  $w_2$  where  $w_1 \neq w_2$ , but the original part-of between  $p_1$  and  $w_1$  remains disabled.

# Life cycles



# Example

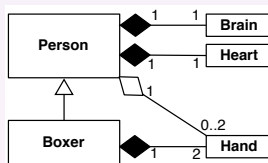
- Need to represent difference between essential vs mandatory vs immutable parts and wholes



- Brain is an **essential part** of Human
- Heart is a **mandatory part** of Human but a heart can be transplanted
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- More generally: the life cycle semantics of parts and wholes

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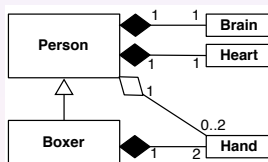


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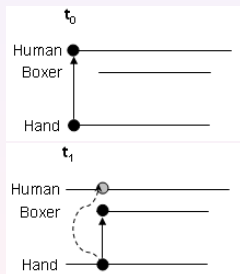
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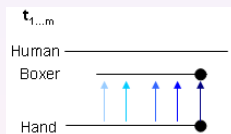
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# The Boxer's hand (with p4)



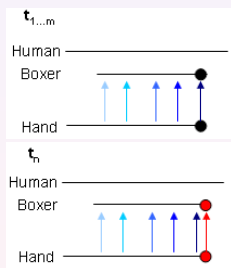
- (1)  $\text{Human} \sqsubseteq \Box^* \text{Human}$  (RIGID)
- (2)  $\text{HumanHand} \sqsubseteq \text{PartWhole}$
- (3)  $\text{HumanHand} \sqsubseteq \text{part} : \text{Hand} \sqcap \text{whole} : \text{Human}$
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- (6)  $\text{Boxer} \sqsubseteq \exists^{=2}[\text{whole}] \text{HumanHand}$  (MANP)
- (7)  $\text{Suspended-HumanHand} \sqsubseteq \perp$  (CONPo)
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- (10)  $\Sigma_{st} \models R \sqsubseteq \Box^+ (R \sqcup \text{Suspended-R} \sqcup \text{Disabled-R})$  (RACT)
- (11)  $\Sigma_{st} \models \text{Disabled-C}_1 \sqcap \Diamond \neg \exists [U_1] R \sqsubseteq \exists [U_1] \text{Disabled-R}$  (RDISAB4)

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# Effect of temporalising on the types of part-whole relations

- Not fully clear—yet.
- Very preliminary results in [AK08].
  - If something is *physically* a *proper* part of a whole it **cannot** be *directly* part of another whole at the same time (idem proper containment and location)
  - proper subprocess **can** participate in different grander processes concurrently, idem members
  - subquantities: (i) for measured amounts of matter of the same type (before/after), and (ii) amounts of different type (concurrently or sequentially)

# Summary

- Disambiguation of types of part-whole relations by means of a taxonomy of types of part-whole relations
- Guidelines for manual modelling
- Reasoning over a relational hierarchy with the RBox Compatibility service
- Representing essential vs mandatory part and wholes with temporal logic ( $D\mathcal{L}\mathcal{R}_{US}$ ) and  $\mathcal{ER}_{\mathcal{VT}}$  extended with status relations

# Discussion

- Part-whole relation as first-class citizen?
- Differences set theory and mereology
  - Mathematical
  - Philosophical (ontological)
- Ease of representation vs accuracy
  - ‘cannot’ include all mereological facets precisely
  - Computational support vs just representing domain knowledge or reality
- “we did just fine without it for decades”
- Better no constructor than a badly defined one?



Artale, A., Franconi, E., Guarino, N., Pazzi, L. (1996). Part-Whole Relations in Object-Centered Systems: an Overview. *Data & Knowledge Engineering*, 1996, 20(3):347-383.



Artale, A., Guarino, N., & Keet, C.M. Formalising temporal constraints on part-whole relations. *11th International Conference on Principles of Knowledge Representation and Reasoning (KR'08)*. Gerhard Brewka, Jerome Lang (Eds.) AAAI Press. Sydney, Australia, September 16-19, 2008.



Artale, A. and Keet, C.M. Essential, mandatory, and shared parts in conceptual data models. In: *Innovations in Information Systems modeling: Methods and Best Practices*. IGI Global, Advances in Database Research Series, Halpin, T.A., Proper, H.A., Krogstie, J. (Eds.).



Artale, A., Parent, C. & Spaccapietra, S. (2007b). Evolving objects in temporal information systems. *Annals of Mathematics and Artificial Intelligence (AMAI)*, 2007, 50(1-2), 5-38.



Bittner, T., Smith, B. A Theory of Granular Partitions. In: *Foundations of Geographic Information Science*, Duckham, M., Goodchild, MF, Worboys, MF (eds.), London: Taylor & Francis Books, 2003, pp117-151.



Bittner, T., Donnelly, M. Computational ontologies of parthood, componenthood, and containment, In: *Proceedings of the Nineteenth International Joint Conference on Artificial Intelligence 2005 (IJCAI05)*. Kaelbling, L. (ed.). pp382-387.



Bittner, T., and Donnelly, M. A temporal mereology for distinguishing between integral objects and portions of stuff. In *Proc. of AAAI'07*, 287–292.



Borgo, S., Masolo, C. Full mereogeometries. *Journal of Philosophical Logic*. (to appear).





Gerstl, P., Pribbenow, S. Midwinters, end games, and body parts: a classification of part-whole relations. *Intl. Journal of Human-Computer Studies*, 1995, 43:865-889.



Guizzardi, G. *Ontological foundations for structural conceptual models*. PhD Thesis, Telematica Institute, Twente University, Enschede, the Netherlands. 2005.



Guizzardi, G. Modal Aspects of Object Types and Part-Whole Relations and the de re/de dicto distinction. *19th International Conference on Advances in Information Systems Engineering (CAiSE)*, Trondheim, Norway, 2007. Springer-Verlag, Berlin, Lecture Notes in Computer Science 4495.



Keet, C.M. Part-whole relations in Object-Role Models. *2nd International Workshop on Object-Role Modelling (ORM 2006)*, Montpellier, France, Nov 2-3, 2006. In: OTM Workshops 2006. Meersman, R., Tari, Z., Herrero., P. et al. (Eds.), Lecture Notes in Computer Science 4278. Berlin: Springer-Verlag, Berlin. pp1116-1127. 2006.



Keet, C.M. *Introduction to part-whole relations: mereology, conceptual modelling and mathematical aspects*. TR KRDB06-3, KRDB Centre, Faculty of Computer Science, Free university of Bozen-Bolzano, Italy. <http://www.inf.unibz.it/krdp/pub/>. 2006.



Keet, C.M. A Formal Theory of Granularity. PhD thesis, KRDB Research Centre, Faculty of Computer Science, Free University of Bozen-Bolzano, Italy. 2008.



Keet, C.M., Artale, A. Representing and reasoning over a taxonomy of part-whole relations. *Applied Ontology*, 2008, 3(1): x-x.



Masolo, C., Borgo, S., Gangemi, A., Guarino, N. and Oltramari, A. (2003). *Ontology Library*. WonderWeb Deliverable D18 (ver. 1.0, 31-12-2003). <http://wonderweb.semanticweb.org>.



Motschnig-Pitrik, R., Kaasbøll, J. Part-Whole Relationship Categories and Their Application in Object-Oriented Analysis. *IEEE Transactions on Knowledge and Data Engineering*, 1999, 11(5):779-797.



Odell, J.J. *Advanced Object-Oriented Analysis & Design using UML*. Cambridge: Cambridge University Press. 1998.



Rector, A., Rogers, J., Bittner, T. Granularity, scale and collectivity: When size does and does not matter. *Journal of Biomedical Informatics*, 2006, 39(3), 333-349.



Sattler, U. A concept language for an engineering application with part-whole relations. In: *DL Workshop*, Borgida, A., Lenzerini, M., Nardi, D., Nebel, B. (Eds.), 1995. pp119-123.



Schulz, S., Hahn, U., Romacker, M. Modeling Anatomical Spatial Relations with Description Logics. *Proceedings of the AMIA Symposium 2000*. Overhage, J.M. (ed.). 2000. pp779-83.



Smith, B., Ceusters, W., Klagges, et al. Relations in biomedical ontologies. *Genome Biology*, 2005, 6:R46.



Varzi, A.C. Mereology. *The Stanford Encyclopedia of Philosophy*. (Fall 2004 Edition), Zalta, E.N. (ed.). <http://plato.stanford.edu/archives/fall2004/entries/mereology/>.



Varzi, A.C. A Note on the Transitivity of Parthood. *Applied Ontology*, 2006, 1(2): 141-146.



Winston, M.E., Chaffin, R., Herrmann, D. A taxonomy of part-whole relations. *Cognitive Science*, 1987, 11(4):417-444.