A taxonomy of part-whole relations

Maria Keet

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Guidelines to choose the appropriate type Reasoning over a hierarchy of relations

Orthogonal subtopics

Representing and reasoning over a taxonomy of part-whole relations

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Orthogonal subtopics

Is a tunnel part of the mountain?

- What is the difference, if any, between how Cell nucleus and Cell are related and how Receptor and Cell wall are related?
- And w.r.t. Brain part of Human and/versus Hand part of Boxer? (assuming boxers must have their own hands)
- A classical example: hand is part of musician, musician part of orchestra, but clearly, the musician's hands are not part of the orchestra. Is part-of then not transitive, or is there a problem with the example?

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Linguistic use of part-whole relations (meronymy)

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- Part of?
 - \star Centimeter part of Decimeter
 - * Decimeter part of Meter
 - therefore Centimeter part of Meter
 - * Meter part of SI
 - but not Centimeter part of SI
- Transitivity?
 - * Person part of Organisation
 - * Organisation located in Bolzano
 - therefore Person located in Bolzano?
 - but *not* Person part of Bolzano

Linguistic use of part-whole relations (meronymy)

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- Orthogonal subtopics

- Which part of?
 - \star CellMembrane structural part of RedBloodCell
 - ★ RedBloodCell part of Blood
 - but not CellMembrane structural part of Blood
 - \star Receptor structural part of CellMembrane
 - therefore Receptor structural part of RedBloodCell

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- Which part of?
 - \star CellMembrane structural part of RedBloodCell
 - * RedBloodCell contained in? Blood
 - but not CellMembrane structural part of Blood
 - \star Receptor structural part of CellMembrane
 - therefore Receptor structural part of RedBloodCell

Part-whole relations in realism-based ontologies

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- Instance-level relations in the OBO Relations Ontology [Seta105]
 - c part_of c₁ at t a primitive relation between two continuant instances and a time at which the one is part of the other
 - *p* **part_of** *p*₁, *r* **part_of** *r*₁ a primitive relation of parthood, holding independently of time, either between process instances (one a subprocess of the other), or between spatial regions (one a subregion of the other)
 - c contained_in c_1 at $t \triangleq c$ located_in c_1 at t and not c overlap c_1 at t
 - c located_in r at t a primitive relation between a continuant instance, a spatial region which it occupies, and a time

Definitions in OBO Relations Ontology [Seta105]

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- Class-level relations
 - C part_of $C_1 \triangleq$ for all c, t, if Cct then there is some c_1 such that C_1c_1t and c part_of c_1 at t.
 - *P* part_of $P_1 \triangleq$ for all *p*, if *Pp* then there is some p_1 such that: P_1p_1 and *p* part_of p_1 .
 - C contained_in $C_1 \triangleq$ for all c, t, if Cct then there is some c_1 such that: C_1c_1t and c contained_in c_1 at t
- Need to commit to a foundational ontology. Recently, linked to BFO http://obofoundry.org/ro/#mappings (test release)
- Same labels, different relata and only a textual constraint: Label the relations differently

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Analysis of the issues from diverse angles

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- Mereological theories [Varzi04], usage & extensions (e.g. mereotopology, relation with granularity, set theory)
- Early attempts with direct parthood [Sattler95], SEP triples [SH00], and other outstanding issues [AFGP96], some still remaining [KA08].
- Cognitive & linguistic issues from meronymy ([WCH87] and so forth)
- Usage in conceptual modelling and ontology engineering
- Subject domains: thus far, mainly geo, bio, medicine

Analysis of the issues from diverse angles

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Ground Mereology

Reflexivity (everything is part of itself)

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Orthogonal subtopics

 $\forall x(part_of(x,x))$

$$\forall x, y ((part_of(x, y) \land part_of(y, x)) \rightarrow x = y)$$
 (2)

(1)

Transitivity (if x is part of y and y is part of z, then x is part of z)

$$\forall x, y, z((part_of(x, y) \land part_of(y, z)) \rightarrow part_of(x, z)) \quad (3)$$
Proper parthood

$$\forall x, y (proper_part_of(x, y) \equiv part_of(x, y) \land \neg part_of(y, x))$$

Ground Mereology

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Orthogonal subtopics

Proper parthood

$$\forall x, y (proper_part_of(x, y) \equiv part_of(x, y) \land \neg part_of(y, x))$$
(5)

Asymmetry (if x is part of y then y is not part of x)

$$\forall x, y(part_of(x, y) \to \neg part_of(y, x))$$
(6)

Irreflexivity (x is not part of itself)

$$\forall x \neg (part_of(x, x)) \tag{7}$$

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Defining other relations with part_of

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Orthogonal subtopics Overlap (x and y share a piece z)

 $\forall x, y(\textit{overlap}(x, y) \equiv \exists z(\textit{part_of}(z, x) \land \textit{part_of}(z, y))) (8)$

Underlap (x and y are both part of some z)

 $\forall x, y (underlap(x, y) \equiv \exists z (part_of(x, z) \land part_of(y, z))) (9)$ Over- & undercross (over/underlap but not part of) $\forall x, y (overcross(x, y) \equiv overlap(x, y) \land \neg part_of(x, y)) (10)$ $\forall x, y (undercross(x, y) \equiv underlap(x, y) \land \neg part_of(y, x))$ (11)

Proper overlap & Proper underlap

 $\forall x, y(p_overlap(x, y) \equiv overcross(x, y) \land overcross(y, x)) (12)$ $\forall x, y(p_underlap(x, y) \equiv undercross(x, y) \land undercross(y, x))$ $(13)_{<0} \land (13)_{<0} \land (13)$

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- With x as part, what to do with the remainder that makes up y?
 - Weak supplementation: every proper part must be supplemented by another, disjoint, part. MM
 - Strong supplementation: if an object fails to include another among its parts, then there must be a remainder.
 EM
- Problem with EM: non-atomic objects with the same proper parts are identical, because of this (extensionality principle), but sameness of parts may not be sufficient for identity E.g.: two objects can be distinct purely based on arrangement of its parts, differences statue and its marble (multiplicative approach)

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General Extensional Mereology

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Orthogonal subtopics Strong supplementation [EM]

 $\neg part_of(y, x) \rightarrow \exists z(part_of(z, y) \land \neg overlap(z, x))$ (14)

And add unrestricted fusion [GEM]. Let \u03c6 be a property or condition, then for every satisfied \u03c6 there is an entity consisting of all entities that satisfy \u03c6. ¹ Then:

 $\exists x \phi \to \exists z \forall y (overlap(y, z) \leftrightarrow \exists x (\phi \land overlap(y, x))) (15)$

- Note that in EM and upward we have identity, from which one can prove acyclicity for ppo
- There are more mereological theories, and the above is not uncontested (more about that later)

¹Need to refer to classes, but desire to stay within FOL. Solution: axiom schema with only predicates or open formulas $\rightarrow (a \rightarrow b) = (a \rightarrow b) =$

Relations between common mereological theories



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Orthogonal subtopics Can any of this be represented in a decidable fragment of first order logic for use in ontologies and (scalable) software implementations?

Things are improving...

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Orthogonal subtopics ■ Early days (90s) and simplest options: DL-role R as partof, or has-part added as primitive role as >, model it as the transitive closure of a parthood relation (16) and define e.g. Car as having wheels that in turn have tires [AFGP96] (17):

$$\succeq \doteq (\texttt{primitive-part}) * \tag{16}$$

$$Car \doteq \exists \succeq . (Wheel \sqcap \exists \succeq . Tire)$$
 (17)

Then Car $\sqsubseteq \exists \succeq$.Tire

■ SEP triples with *ALC* [SHO0]

 What SHIQ fixes cf. ALC: Transitive roles, Inverse roles (to have both part-of and has-part), Role hierarchies (e.g. for subtypes of part-of), qualified Number restrictions (e.g. to represent that a bycicle has-part 2 wheels)

Build-your-own DL-language [BD05]

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What we can(not) implement now with DL-based ontology languages

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Orthogonal subtopics

Table: Properties of parthood and proper parthood compared to their support in \mathcal{DLR}_{μ} , \mathcal{SHOIN} and \mathcal{SROIQ} . *: properties of the parthood relation (in M); [‡]: properties of the proper parthood relation (in M).

$Language \Rightarrow$	\mathcal{DLR}_{μ}	SHOIN	SROIQ	DL-Lite _A
Feature ↓		(\sim OWL-DL)	(\sim OWL 2)	
Reflexivity *	+	-	+	_
Antisymmetry *	-	-	-	-
Transitivity * [‡]	+	+	+	-
Asymmetry [‡]	+	+	+	+
Irreflexivity ‡	+	-	+	-
Acyclicity	+	-	_	_

Taxonomy of types of part-whole relations

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- Clarifying and cleaning up ontologically the informal usages of "part of"
- Sorting out (in)transitivity and why some part-whole relations are non- or in-transitive, but parthood is transitive

Addressing the issues

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- Efforts to disambiguate the confusion on part-whole relations; e.g. an informal taxonomy [WCH87], list of 6 types motivated by conceptual modelling [Odel198] [GP95], and ontology-inspired conceptual modelling [G05]
- Location, containment, membership of a collective, quantities of a mass
- Relatively well-settled debate on transitivity, or not [Varzi06] and related papers in AO

Overview

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Guidelines to choose the appropriate type Reasoning over a hierarchy of relations

- Mereological part_of (and subtypes) versus 'other' part-whole relations
- Categories of object types of the part-whole relation changes
- Structure these relations by (non/in)transitivity and kinds of relata
- Simplest mereological theory, **M**.
- Commit to a foundational ontology: DOLCE [MBGG003] (though one also could choose, a.o., BFO, OCHRE, GFO, ...)

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DOLCE categories





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Orthogonal subtopics "member-bunch", collective nouns (e.g. Herd, Orchestra) with their members (Sheep, Musician)

Definition 1 (member_of)

 $\forall x, y (member_of_n(x, y) \triangleq mpart_of(x, y) \land (POB(x) \lor SOB(x)) \land SOB(y))$

"material-object", that what something is made of (e.g., Vase and Clay)

Definition 2 (constituted_of)

 $\forall x, y (constitutes_{it}(x, y) \equiv constituted_of_{it}(y, x) \triangleq mpart_of(x, y) \land POB(y) \land M(x))$

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Orthogonal subtopics "quantity-mass", "portion-object", relating a smaller (or sub) part of an amount of matter to the whole. Two issues (glass of wine & bottle of wine vs. Salt as subquantity of SeaWater)

Definition 3 (sub_quantity_of)

 $\forall x, y (sub_quantity_of_n(x, y) \triangleq mpart_of(x, y) \land M(x) \land M(y))$

"noun-feature/activity", entity participates in a process, like Enzyme that participates in CatalyticReaction

Definition 4 (participates_in)

 $\forall x, y (participates_{init}(x, y) \triangleq mpart_of(x, y) \land ED(x) \land PD(y))$

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Orthogonal subtopics Object and its 2D or 3D region, such as contained_in(John's address book, John's bag) and located_in(Galway, Ireland)

Definition 5 (contained_in)

 $\forall x, y (contained_in(x, y) \triangleq part_of(x, y) \land R(x) \land R(y) \land \\ \exists z, w (has_3D(z, x) \land has_3D(w, y) \land ED(z) \land ED(w)))$

Definition 6 (located_in)

 $\forall x, y (located_in(x, y) \triangleq part_of(x, y) \land R(x) \land R(y) \land \\ \exists z, w (has_2D(z, x) \land has_2D(w, y) \land ED(z) \land ED(w)))$

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Orthogonal subtopics processes and sub-processes (e.g. Chewing is involved in the grander process of Eating)

Definition 7 (involved_in)

 $\forall x, y (involved_{in}(x, y) \triangleq part_of(x, y) \land PD(x) \land PD(y))$

extensibility, e.g.

Definition 8 (s_part_of)

 $\forall x, y(s_part_of(x, y) \triangleq part_of(x, y) \land ED(x) \land ED(y))$

Using the taxonomy of part-whole relations

A taxonomy of part-whole relations

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- Guidelines to choose the appropriate type Reasoning over a hierarchy of relations
- Orthogonal subtopics

- Representing it correctly in ontologies and conceptual data models
- Reasoning with a taxonomy of relations

Decision diagram [K06a]



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Decision diagram



Decision diagram



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Orthogonal subtopics



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Example - before



Orthogonal subtopics

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- Envelope is not involved-in, not a member-of, does not constitute, is not a sub-quantity of, does not participate-in, is not a geographical object, but instead is contained-in the ConferenceBag.
 - Transitivity holds for the mereological relations: derived facts are automatically correct, like RegistrationReceipt contained-in ConferenceBag.
- Intransitivity of Linen and ConferenceBag, because a conference bag is not wholly constituted of linen (the model does not say what the Flap is made of).
- Completeness, i.e. that *all* parts make up the whole, is implied thanks to the closed-world assumption.
 ConferenceBag directly contains the ConfProceedings and Envelope only, and does not contain, say, the Flap.

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Orthogonal subtopics

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Requirements

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Using the taxonomy of part-whole relations

Guidelines to choose the appropriate type Reasoning over a hierarchy of relations

- Represent at least Ground Mereology,
- Express ontological categories and their taxonomic relations,
- Having the option to represent transitive and intransitive relations, and
- Specify the domain and range restrictions (/relata/entity types) for the classes participating in a relation.

Current behaviour of reasoners

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Orthogonal subtopics

A1. Class hierarchy with asserted conditions





B. Correct role box (object properties)



C. Wrong role box (object properties)



Current behaviour of reasoners

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Orthogonal subtopics

1. A1+B+racer: ontology OK

3. A1+C+racer: class hierarchy is inconsistent

- Theck concept consistency
 - Time to build query = less that 0.001 seconds
 - Time to send and receive from reasoner = 0.031 seconds
 - ----- Inconsistent concepts

🛞 Chassis is inconsistent

----- Time to update Protege-OWL = 0.016 seconds

2. A2+B+racer: ontology OK

4. A2+C+racer: Chassis reclassified as PD

Computing superclasses: Querying reasoner...



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The RBox Compatibility service – definitions

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Orthogonal subtopics

Definition 9 (Domain and Range Concepts)

Let *R* be a role and $R \sqsubseteq C_1 \times C_2$ its associated Domain & Range axiom. Then, with the symbol D_R we indicate the *User-defined Domain* of *R*—i.e., $D_R = C_1$ —while with the symbol R_R we indicate the *User-defined Range* of *R*—i.e., $R_R = C_2$.

Definition 10 (RBox Compatibility)

For each pair of roles, R, S, such that $\langle \mathcal{T}, \mathcal{R} \rangle \models R \sqsubseteq S$, check: Test 1. $\langle \mathcal{T}, \mathcal{R} \rangle \models D_R \sqsubseteq D_S$ and $\langle \mathcal{T}, \mathcal{R} \rangle \models R_R \sqsubseteq R_S$;

Test 2.
$$\langle \mathcal{T}, \mathcal{R} \rangle \not\models D_S \sqsubseteq D_R$$

Test 3.
$$\langle \mathcal{T}, \mathcal{R} \rangle \not\models R_S \sqsubseteq R_R$$
.

An RBox is said to be compatible iff *Test* 1 and (2 or 3) hold for all pairs of role-subrole in the RBox. 53/68

The RBox Compatibility service - behaviour

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Orthogonal subtopics

- If Test 1 does not hold: warning that domain & range restrictions of either R or S are in conflict with the role hierarchy proposing either
 - (i) To change the role hierarchy or
 - (ii) To change domain & range restrictions or
 - (iii) If the test on the domains fails, then propose a new axiom $R \sqsubseteq D'_R \times R_R$, where $D'_R \equiv D_R \sqcap D_S^2$, which subsequently has to go through the RBox compatibility service (and similarly when Test 1 fails on range restrictions).

²The axiom $C_1 \equiv C_2$ is a shortcut for the axioms: $C_1 \equiv C_2$ and $C_2 \equiv C_{D,Q,Q}$

The RBox Compatibility service - behaviour

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- If Test 2 and Test 3 fail: warn that R cannot be a proper subrole of S but that the two roles can be equivalent. Then, either:
 - (a) Accept the possible equivalence between the two roles or
 - (b) Change domain & range restrictions.
- Ignoring all warnings is allowed, too

Extensions in various directions

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- Mereotopology, with location, GIS, Region Connection Calculus (http://www.comp.leeds.ac.uk/qsr/rcc.html)
 - Mereogeometry [ВМ07]
 - Mereology and/vs granularity (a.o. [BS03], [K08], [RRB06])
 - Temporalising the part-whole relations (a.o. [BD07] [AGK08])

Some more issues in parthood relations

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Orthogonal subtopics

- What to do with other (meta-)properties, such as:
 - Inseparability (x is inseparable of y iff x is existentially dependent on y and x is necessarily a part of y, see [G05]
 p169)
 - Degrees of shareability ('total', part of more than whole of the same type or of different types, etc. [MK99])
 - Essential part & essential whole (like member-partnership [Odel198], brain-human)
 - Immutable part & immutable whole (the boxer with his hands, an ecofarm with a piece of farmland)

■ **De dicto/de re** distinction (possible worlds) [G07], where - de re with $\forall ... \rightarrow \Box ..., e.g.$ "every boxer necessarily has a hand"

- de dicto with $\Box(orall...
ightarrow...)$ e.g. "necessarily, every boxer has a hand"

Total/complete on the parts

■ **Direct part** to distinguish from part-by-transitivity

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Some more issues in parthood relations

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Orthogonal subtopics

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 - de re with $\forall ... \rightarrow \Box ..., e.g.$ "every boxer necessarily has a hand"
 - de dicto with $\Box(orall ...
 ightarrow ...)$ e.g. "necessarily, every boxer has a hand"
- Total/complete on the parts
- Direct part to distinguish from part-by-transitivity

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(Non-)Sharable parts and wholes

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Guidelines to choose the appropriate type Reasoning over a hierarchy of relations

- Essential and mandatory parts; informally, distinguish between:
 - "must be part of *some* whole" and
 - "must be part of *the same* whole" during its lifetime.
- Sequentially or concurrently being part of > 1 whole of same or different type; distinguish between, a.o.:
 - p is part of $w_1 \in W$ at time t_1 and of $w_2 \in W$ at time t_2 ;
 - *p* is part of $w_1 \in W$ and $w_2 \in W$ at time t_1 ;
 - p is part of $w_1 \in W$ at time t_1 and of $w_a \in W'$ at time t_2 ;
 - *p* is part of $w_1 \in W$ and $w_a \in W'$ at time t_1 ;

Solution sketch

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- Temporalizing part-whole relations, and parts and wholes
 - $\mathcal{DLR}_{\mathcal{US}}$ and $\mathcal{ER}_{\mathcal{VT}}$ [APS07], extended with status relations
 - Details in [AGK08]

Examples

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- Scheduled: a relation is scheduled if its instantiation is known but its membership will only become effective some time later. For instance, a pillar for finishing the interior of the Sagrada Familia in Barcelona is scheduled to become part of that church.
- Active: the status of a relation is active if the particular relation fully instantiates the type-level relation. For instance, the Mont Blanc mountain is part of the Alps mountain range, and the country Republic of Ireland is part of the European Union.

Examples

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- Suspended: to capture a temporarily inactive relation.
 For example, an instance of a CarEngine is removed from the instance of a Car it is part of, for purpose of maintenance at the car mechanic.
- **Disabled**: to model expired relations that never again can be used. For instance, to represent the donor of an organ who has donated that organ and one wants to keep track of who donated what to whom: say, the heart p_1 of donor w_1 used to be a structural part of w_1 but it will never be again a part of it. The heart, p_1 , then may have become participant in a new part-of relation with a new whole, w_2 where $w_1 \neq w_2$, but the original part-of between p_1 and w_1 remains disabled.

Effect of temporalising on the types of part-whole relations

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- Not fully clear—yet.
- Very preliminary results in [ΑΚΟ8].
 - If something is *physically* a *proper* part of a whole it cannot be *directly* part of another whole at the same time (idem proper containment and location)
 - proper subprocess can participate in different grander processes concurrently, idem members
 - subquantities: (i) for measured amounts of matter of the same type (before/after), and (ii) amounts of different type (concurrently or sequentially)

Summary

A taxonomy of part-whole relations

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- Disambiguation of types of part-whole relations by means of a taxonomy of types of part-whole relations
- Guidelines for manual modelling
- Reasoning over a relational hierarchy with the RBox Compatibility service
- A list of other part-whole topics, such as temporal, mereotopological, modality

Discussion

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- Part-whole relation as first-class citizen?
- Differences set theory and mereology
 - Mathematical
 - Philosophical (ontological)
- Ease of representation vs accuracy
 - 'cannot' include all mereological facets precisely
 - Computational support vs just representing domain knowledge or reality
- "we did just fine without it for decades"
- Better no constructor than a badly defined one?

References

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Orthogonal subtopics

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